Induction of decision trees in numeric domains using set-valued attributes

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Abstract

Conventional algorithms for decision tree induction use an attribute-value representation scheme for instances. This paper explores the empirical consequences of using set-valued attributes. This simple representational extension, when used as a pre-processor for numeric data, is shown to yield significant gains in accuracy combined with attractive build times. It is also shown to improve the accuracy for the second best classification option, which has valuable ramifications for post-processing. To do so an intuitive and practical version of pre-pruning is employed. Moreover, the implementation of a simple pruning scheme serves as an example of pruning applicability over the resulted trees and also as an indication that the proposed discretization *absorbs* much of pruning potential. Finally, we construct several versions of the basic algorithm to examine the value of every component that comprises it.

Keywords

Decision trees, discretization, set-values, post-processing.

1 Introduction

Describing instances using attribute-value pairs is a widely used practice in the machine learning and pattern recognition literature. In this paper we aim to explore the extent to which using such a representation for decision tree learning is as good as one would hope for. Specifically, we will make a departure from the attribute-single-value convention and deal with set-valued attributes.

We will use set-valued attributes to enhance our ability to effectively pre-process numeric variables. Pre-processing attributes will also enhance the capability of deploying promising post-processing schemes: by reporting the second best choice during classification (with increased confidence), one can evaluate alternatives in a sophisticated fashion. In doing so, we face all guises of decision tree learning problems. First of all, we re-define splitting criteria. Then, we argue on the value of pre-pruning, as a viable and reasonable pruning strategy with the set-valued attribute approach. We proceed by altering the classification task to suit the new concepts.

The first and foremost characteristic of set-valued attributes is that during splitting, an instance may be instructed to follow *both* paths of a (binary) decision tree, thus ending up in more than one leaf. Such instance replication across tree branches should improve the quality of the splitting process as splitting decisions are based on larger instance samples. The idea is, then, that classification accuracy should increase. This increase will be effected not only by more educated splits during learning but also because instances follow more than one path during testing too. This results in a combined decision on class membership.

Such instance proliferation, across and along tree branches can look like a problem. At some nodes, it may be that attributes are not informative enough for splitting and that the overlap in instances between children nodes may be excessive. Defining what may be considered excessive has been a line of research in this work and it has turned out that very simple pre-pruning criteria suffice to contain this excessiveness.

As probabilistic classification has inspired this research, we would like to give credit, for some of the ideas that come up in this paper, to the work carried out by Quinlan [16]. However, we emphasize that the proposed approach does not relate to splitting using feature sets, as employed by C4.5 [18].

Option decision trees -as they were modified by Kohavi and Clayton [10]- are quite close to our work. Kohavi and Clayton use a single structure for voting that is easier to interpret than *bagging* [3] and *boosting* [7] which use a set of unrelated trees. Still, our view of the problem is quite different, adopting set-values as the main reason for uncertainty, during both tree creation and classification.

Kohavi and Sahami [9] compared different global discretization methods that used error-based and entropy-based criteria. They stated "*naïve discretization of the data can be potentially disastrous for data mining as critical information may be lost due to the formation of inappropriate bin boundaries*". We believe that the set-valued approach offers better boundaries for the discretization problem by carefully introducing a fuzziness factor.

An earlier attempt to capitalize on these ideas appeared in [8]. Probably due to the immaturity of those results and to the awkwardness of the terminology used therein, the potential has not been fully explored. Of course, this research flavor has been apparent in [13], who discuss overlapping concepts, in [1] and in [12]; the latter two are oriented towards the exploitation of multiple models.

Our short review would be incomplete without mentioning RIPPER [5], a system that exploits set-valued features to solve categorization problems in the linguistic domain. Cohen emphasized on symbolic set-valued attributes, while we *mainly* concentrate on artificially created set-values for numeric attributes. Nevertheless, our approach is readily adaptable to symbolic set-valued domains.

The rest of this paper is organized in six sections. The next section elaborates on the intuition behind the set-valued approach and how it leads to the heuristics employed. The actual modifications to the standard decision tree algorithms come next, including descriptions of splitting, pruning and classifying. We then demonstrate via an extended experimental session that the proposed heuristics indeed work and identify potential pitfalls. The next two sections deal with modifications to the standard algorithm so as to ensure the value of every one of its building blocks. The first one presents variations that use set-values *just* for training or testing while the second builds on the idea of weighted distribution of multiple instances. Finally, we put all the details together and discuss lines of research that have been deemed worthy of following.

2 An Overview of the Problem

Symbolic attributes are usually described by a single value ("Ford is the make of this car"). A symbolic attribute may be characterized by a specific value as a result of the discrete nature of the domain. On the other hand, numeric values are drawn from a continuum of values, where errors or variation may turn up in different guises of the same underlying phenomenon: noise. Note that this can hold of seemingly symbolic values too; colors correspond to a de facto discretization of frequencies. It would be plausible that for some discretization, for a learning problem, we would like to be able to make a distinction between values such as green, not-so-green, etc. but drawing the line could be really difficult. This explains why discretizing numeric values is a practice that easily fits the set-valued attributes context.

A casual browsing of the data sets in the Machine Learning repository [2] shows that there are no data sets where the concept of set-valued attributes could apply in a purely symbolic domain. It comes as no surprise that such experiments are limited and the only practically justifiable ones have appeared in a linguistic context (in robust or predictive parsing, for example, where a word can be categorized both as a verb and a noun, and only semantic context can make the difference) [14].

One of the typical characteristics of decision trees is that their divide-and-conquer approach quickly trims down the availability of large instance chunks near the tree fringe. The splitting process calculates some level of (some type of) information gain and splits the tree accordingly. In doing so, it forces instances that are near a splitting threshold to follow one path. The other one, even if missed by some small *Äå* is a loser. This amounts to a reduced sample in the losing branch. By treating numeric attributes as set-valued ones we artificially enlarge the learning population at critical nodes. This transformation can be done with a discretization step, before any splitting starts. It also means that threshold values do not have to be re-calculated using the actual values but the substituted discrete ones (actually, set of values, where applicable).

During testing, the same rule applies and test instances may end up in more than one leaf. Based on the assumption that such instance replication will only occur where there is doubt about the existence of a single value for an attribute, it follows that one can determine class membership of an instance by considering all leaves it has reached. This delivers a more educated classification as it is based on a larger sample and, conceptually, resembles an ensemble of experts.

3 Learning with Set Valued Attributes

To obtain set-valued attributes we have to discretize raw data. The discretization step produces instances that have (integer) set-valued attributes. Our algorithm uses these normalized instances to build the tree. Every attribute's values are mapped to integers. Instances sharing an attribute value are said to belong to the same bucket, which is characterized by that integer value. Each attribute has as many buckets as distinct values.

For all continuous (numeric) attributes we split the continuum of values into a small number of non-overlapping intervals with each interval assigned to a bucket (one-to-one correspondence). An instance's value (a point) for a specific attribute may then belong to more that one of these buckets. Buckets may be merged when values so allow. Missing values are directed by default to the first bucket for the corresponding attribute.[1]

We use the classic information gain [15] metric to select which attribute value will be the test on a specific node. An attribute's value that has been used is excluded from being used again in any sub-tree. Every instance follows at least one path from the root of the tree to some leaf. An instance can follow more than one branch of a node when the attribute being examined at that node has two values that direct it to different branches. Thus, the final number of instances on leaves may be larger than the starting number of instances.

An interesting side-effect of having instances following both branches of a tree is that a child node can have exactly the same instances with its father. Although this is not necessarily a disadvantage –since more instances can lead to a better splitting decision- a repeating pattern of this behavior along a path can cause a serious overhead due to the size of the resulting tree.

These considerations lead to a simple pre-pruning technique. We prune the tree at a node (we make that node a leaf) when that node shares the same instance set with *some* predecessors (see Figure 1). Quantifying the *some* term is an ad hoc policy, captured by the notion of the pruning level (and thus allowing flexibility). So, for example, with a pruning level of 1 we will prune the tree at a node that shares the same instance set with its father but not with its grandfather. We expect that any information (lost) is likely contained in some of the surviving (replicated) instance sets.

[1] Admittedly this approach is counterintuitive, yet quite straightforward. Our research agenda suggests that missing values instance should be directed to *all* buckets.

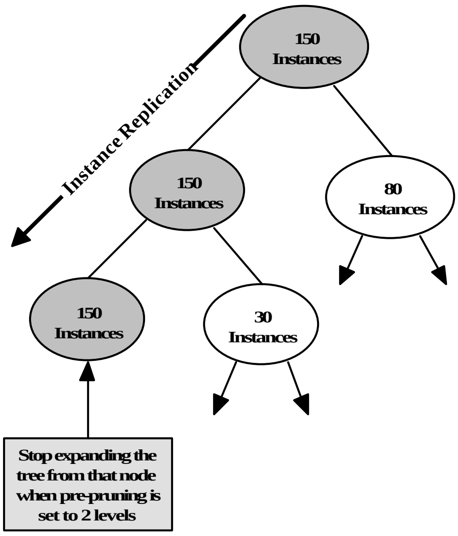

Figure 1: Pre-pruning example

Quite clearly, using the proposed method may well result in testing instances that assign their values to more than one bucket. Thus, the classification stage requires an instance to be able to follow more than one branch of a node ending up, maybe, in more than one leaf. Classification is then straightforward by averaging the instance classes available at all the leaves reached by an instance.

An algorithm that uses the $\div^2$ metric, ChiMerge [11], has been used to discretize continuous attributes. ChiMerge employs a $\div^2$-related threshold to find the best possible points to split a continuum of values.

The value for $\div^2$-*threshold* is determined by selecting a desired significance level and then using a table to obtain the corresponding $\div^2$ value (obtaining the $\div^2$ value also requires specifying the number of *degrees of freedom,* which will be 1 less than the number of classes). For example, when there are 3 classes (2 degrees of freedom) the $\div^2$ value at the .90 percentile level is 4.6. The meaning of $\div^2$-*threshold* is that among cases where the class and attribute are independent there is a 90% probability that the computed $\div^2$ value will be less than 4.6; $\div^2$ values in excess of this threshold imply that the attribute and the class are not independent. Choosing higher values for $\div^2$-*threshold* causes the merging process to continue longer, resulting in fewer and larger intervals (buckets). The user can override $\div^2$-*threshold* by setting a *max-buckets* parameter, thus specifying an upper limit on the number of intervals to create.

During this research we extended the use of the $\div^2$ metric to create left and right margins extending from a bucket's boundaries. Every attribute's value belonging to a specific bucket but also belonging to the left (right) margin of that bucket, was also considered to belong to the previous (next) bucket respectively.

To define a margin's length -for example a right margin- we start by constructing a test bucket, which initially has only the last value of the current bucket and test it with the next bucket as a whole using the $\div^2$ metric. While the result does not exceed $\div^2$-*threshold*, we extend the right margin to the left, thus enlarging the test bucket. We know (from the initial bucket construction) that the right margin will increase finitely and within the bucket's size.

For example, suppose we have an attribute named *whale-size*, which represents the length of, say, 50 whales. To create buckets we sort the values of that attribute in ascending order and we use ChiMerge to split the continuum of values to buckets (see Figure 2).

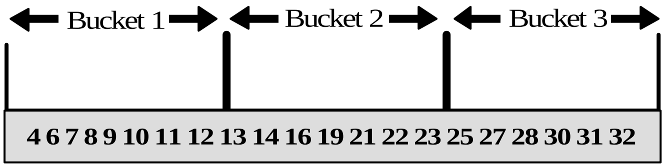

Figure 2: Splitting the continuum of values to buckets

We then move to find the margins. The first bucket has only a right margin and the last has only a left margin. Consider bucket 2 on the figure below; its boundary values are 13 and 23 and its left and right margin are at values 16 and 22 correspondingly.

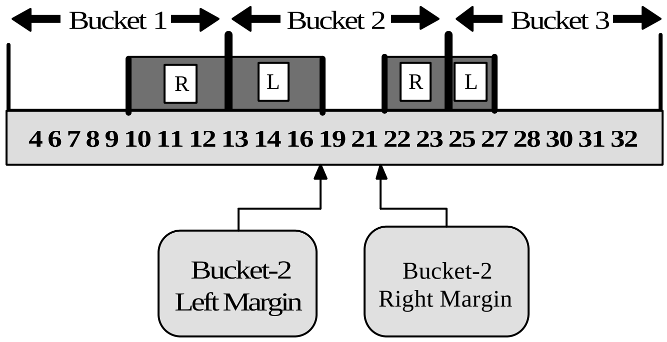

Figure 3: Getting margins for every bucket

To obtain the right margin we start by testing value 23 with bucket 3 as a whole using the $\div^2$ metric. Suppose that the first test returns a value lower than $\div^2$-*threshold*; we set 23 as the right margin of bucket 2 and combine 23 with 22 (the next-to-the-left value of bucket 2) to a test bucket.[2] This test bucket is tested again with bucket 3 using the $\div^2$-statistic. Suppose that, once more, the returned value does not exceed $\div^2$-*threshold*; we extend bucket's 2 right margin to 22. If the next attempt to extend the margin fails, because $\div^2$-*threshold* is exceeded, from then on, when an instance has the value 22 or 23 for its *whale-size* attribute, this value is mapped to both buckets 2 and 3.

4 Experimental Validation

Experimentation consisted by using several databases from the Machine Learning Repository [2]. We compared the performance of the proposed approach to the results obtained by experimenting with ITI [19] as well. ITI is a decision tree learning program that is freely available for research purposes. All databases were chosen to have continuous valued attributes to demonstrate the benefits of our approach. Experimentation took place on a PentiumII 233 running Linux.

The main goal during this research was to observe how the proposed modifications, compared with existing algorithms, would affect (hopefully increase) the accuracy over unseen data. A parallel

[2] If the very first test were unsuccessful we would set the left margin to be the mid-point between bucket's 3 first number (25) and bucket's 2 last number (23), which is 24.

goal was to build a time-efficient algorithm that outputs reasonably sized -and thus comprehensible- decision trees.

As the algorithm depends on a particular configuration of $\div^{2}$*-thresholds, maximum number of buckets* and *pruning level,* we conducted our testing for various combinations of those parameters. During the experimental section we chose to 'lock' *maximum number of buckets* to 15, because more buckets produced significant increase in time spent for the program to execute without showing any accuracy increase[3]. On the other hand less buckets tended to produce unpredictable results; sometimes better and sometimes worse. Furthermore, Kerber suggests that his algorithm should be used with a moderate number of buckets, in the range of [5,15].

Specifically, every database was tested using $\div^{2}$*-threshold* values of 0.90, 0.95, 0.975 and 0.99, with a pruning level of 1, 2, 3, and 4, thus resulting in 16 different tests for every database.

These tests would be then compared to a single run of ITI. Note that each such test is the outcome of a 10-fold cross-validation process.

The following databases were used:

Database	Characteristics	Classes
Abalone	4177 instances, 8 attributes (one nominal), no missing values	3
Adult	48842 instances, 14 attributes (8 nominal), missing values exist	2
Crx	690 instances, 15 attributes (6 nominal), no missing values	2
Ecoli	336 instances, 8 attributes (one nominal), no missing values	8
Glass	214 instances, 9 attributes (all numeric), no missing values	6
Pima	768 instances, 8 attributes (all numeric), no missing values	2
Wine	178 instances, 13 attributes (all numeric), no missing values	3
Yeast	1484 instances, 8 attributes (one nominal), no missing values	10

Minimum-error-pruning (MEP) was adopted, in order to demonstrate the applicability of post-pruning over the resulted trees. This pruning method suffers from a series of disadvantages, the most important one being the dependence of pruning extent on the number of classes. Furthermore, it is not clear yet, whether a pruning algorithm should be used at its original form, due to the complicated maths that are induced by the replicated instances. Nevertheless, MEP's simplicity combined with our intention not to embark –for the time being- to a full pruning research project convinced us to adopt it.

Therefore, due to the immaturity of the pruning implementation, the proposed algorithm has to be compared ***mainly*** with un-pruned ITI. Our pre-pruning method ought to be viewed as a slightly complicated way to stop building the tree – almost every decision tree algorithm uses such stopping criteria, usually when possible test attributes are exhausted. Yet, the proposed algorithm surpasses un-pruned ITI while at the same time being very competitive to its pruned version.

Two metrics were recorded: accuracy and size. The results are presented at the Appendices. Accuracy is the percentage of the correctly classified instances, and size is the number of leaves of the resulting decision tree. Although speed -the time that the algorithm spends to create the tree- is not reported, it was proportional to the time spent by ITI. Our algorithm recouped the time spent during the discretization step through less expensive manipulations of integers during tree creation and classification. It has been noted by Catlett [4] that for very large data sets (as is common in real-world data mining applications) discretizing continuous features can often vastly reduce the time necessary to induce a classifier.

[3] In fact, most datasets' accuracy slightly decreased with unlimited number of buckets.

As stated earlier in this paper, one of the goals was to demonstrate that the proposed modifications improve not only accuracy in the conventional sense, but also our ability to generate good second- or third-place alternatives, for use in post-processors. We term this extension *Coverage*: reporting for a coverage of 2 means that we report the accuracy when we allow the real class to be among the best 2 returned by the classifier. ITI had to be modified to report *Coverage*; this way we were able to compare its outcomes with the set-valued results.

Although experiments were conducted both for *Coverage-2* and *Coverage-3* we chose to present only *Coverage-2* outcomes due to their more valuable repercussions. Note that coverage k results for data sets of k classes is necessarily 100%. Experiments have not been conducted when this was the case (*Adult*, *Crx*).

We felt that a graphical representation of the relative superiority of the one or the other algorithm would be more emphatic, so we chose a three-dimensional method for presenting them. This way we were hoping to unveil hidden characteristics of the whole approach; identifying good points or possible pitfalls. Throughout the experiments, x-axis represents $\div^{2}$*-threshold* and y-axis represents the *pruning level*. On the other hand, z-axis represents accuracy or size differences between the two different algorithms, noted at the top of each page.

We first present the accuracy results (Appendix A). The comparison of un-pruned trees gives a clear winner; with the exception of *Glass* database the proposed approach outperforms ITI by a comfortable margin. It is interesting to note that under-performance (regardless of whether this is observed against ITI with or without pruning) is usually associated with high $\div^{2}$*-threshold* values combined with small pruning levels – that is the case for *Glass, Pima and Yeast*. Worse results for small pruning levels are an indication that instance replication is actually beneficial. Furthermore, increasing $\div^{2}$*-threshold* tends to produce steeper lines in conjunction with pruning level. The $\div^{2}$-threshold metric alone does not seem to produce consistent results throughout the experiments. It rather seems that every database, depending on its idiosyncrasy, prefers more or less fuzziness during the creation of buckets. Finding a specific combination of these parameters under which our approach does best for all circumstances is a difficult task and, quite surely, theoretically intractable.

It is interesting to compare our **un-pruned** trees with ITI's **pruned** ones. This comparison indicates a better performance for our approach under *Abalone*, *Ecoli*, *Wine* and *Yeast* and a worse one under *Adult*, *Glass*, *Pima* and *Crx*. This is extremely promising regarding the possibilities of the proposed heuristic; we believe that a sophisticated pruning scheme, such as ITI's approach which is based on minimum-description-length principle (MDL) [17], can produce even better results.

The simple pruning strategy currently in use gave ambiguous results, compared to the results of the unpruned trees- with the exception of *Glass* database where pruning results are clear worse[4]. Still, the differences are not great indicating that this kind of pruning just did not offer much to the direction of trees generalization. We conjecture that our algorithm ***absorbed*** much of the pruning potential.

Coverage results (see Appendix B) are almost throughout consistent indicating a higher accuracy for the proposed approach. *Pima* database was the only one to return a higher accuracy for ITI. Recall that, ITI stops building the tree when all instances at a node belong to the same class. Thus, un-pruned ITI could not show any improvement for *Coverage-2* – it simply did not have any other class to return. Pruning changes that, since a newly created leaf originates from a node and thus its instances should belong to more than one class.

4 Recall that, Glass database has only 214 instances. Furthermore, Glass has six possible problem classes, something that does not favor minimum-error-pruning.

The main drawback of the algorithm is the size of the trees that it constructs (see Appendix C). All graphs show a clear increase of trees size in conjunction with pruning level. On the other hand, results for $\div^2$-*threshold* are not –for once more- consistent throughout the experiments. Having instances replicate themselves can result in extremely large, difficult to handle trees. Two things can prevent this from happening: discretization reduces the size of the data that the algorithm manipulates and pre-pruning reduces the unnecessary instance proliferation along tree branches. We have seen that both approaches seem to work well, yet none is a solution per se. Pruning helps on this direction too. Pruned trees are of reasonable size; sometimes even smaller than the pruned trees of ITI. Still, *adult* database produced a significant bigger tree, even when we had it pruned.

5 Training and Testing Policies

The cautious reader may question the effectiveness of the set-valued approach. The observed accuracy increase could be due to the discretization step itself and thus it could be attributed just to the $\div^2$ metric and not to the set-values extension. Other questions could be raised as well, such as, why not to use set-valued instances *just* during training or testing – and thus preserve computing resources. Our research would be surely incomplete without taking these questions into consideration.

For those reasons we devised several training and testing policies. The first one uses the $\div^2$ metric to create buckets but avoids the formation of margins. This way, every attribute-value is mapped to a single bucket, and that stands for both training and testing instances (we call it *single-training/single-testing* in contrast to our main *multi-training/multi-testing* algorithm). The derived algorithm can be considered as a standard decision tree creation algorithm -not very different in philosophy from ITI- that uses ChiMerge to discretize data.

The second derived algorithm does form margins, but uses them just for training instances. All testing instances have their attribute-values assigned to only one bucket. We call this version *multi-training/single-testing*. Using the same line of thought, we derive the third version of the algorithm which is the exact dual of the previous one - it uses margins just for testing instances. This version is called *single-training/multi-testing*.

The derived algorithms produce different classes of solutions for the build/use of decision trees, with most of the known algorithms (like ID3, C4.5, ITI) belonging to the *single-training/single testing* class. An easy way to distinguish between those classes is to draw the paths that an instance follows during training and testing (Figure 4).

Having introduced the modified algorithms we embark into an introspection journey through their advantages and disadvantages. When we compare ITI with our *single-training/single-testing* algorithm[5] (Appendix D_1) we come up with a higher accuracy for the $\div^2$ discretization method for the majority of $\div^2$-*thresholds* and databases. This is a strong experimental evidence in favour of ChiMerge. Still, as we shall see, we can do even better.

The comparison between the *single-training/single-testing* and *multi-training/multi-testing* algorithm (Appendix D_2) provide us with a clear indication that the set-valued approach adds *something* to the learning process that can be exploited towards higher accuracy levels. This exploitation can be achieved using the notion of pruning level. It is clear, under every $\div^2$-*threshold* level, that an increase in the pruning is associated with some increase in accuracy. It is also clear that when an increase in the introduced fuzziness (by increasing $\div^2$-*threshold*) destabilizes the system (and thus, produces bad accuracy results) the pruning level reacts as a stabilising factor (producing steeper improvement lines) which quickly fills the performance gap. It seems likely that

[5] All comparisons are between unpruned version of trees.

where standard algorithms achieve an accuracy of *a%*, the set-valued approach has a potential range of performance between [$a\text{-}\mathring{a}_1$, $a+\mathring{a}_2$]. It is the pruning level that gets us closer to the right limit.

Appendix D_3 presents the accuracy comparison between the *multi-training/multi-testing* and the *multi-training/single-testing* algorithms. It is obvious that when we avoid set-values during testing (and thus reach only one leaf) we come up with bad results. This can be attributed to the absence of the majority scheme vote, which seems to make good use of the introduced fuzziness.

The results of Appendix D_4 (*multi-training/multi-testing* vs *single-training/multi-testing)* were somewhat unexpected. Those results indicate a higher accuracy, over half of the tested databases, for the version of the heuristic that uses margins *just* during testing. Recall that this version can be viewed as an algorithm that uses a fuzzy-thresholds-technique to guide test-instances; such techniques are widely used in current algorithms. The fact that this algorithm does well was expected, although not to such a degree. Still, there are databases (*Wine*, *Ecoli*, *Glass*) where the *multi-training/multi-testing* heuristic surpasses the *single-training/multi-testing* algorithm by a more than marginal accuracy difference. Further experiments have to be conducted towards this direction, in order to establish the class of problems/databases that can be assisted *more* by the *multi-training/multi-testing* than by the *single-training/multi-testing* algorithm.

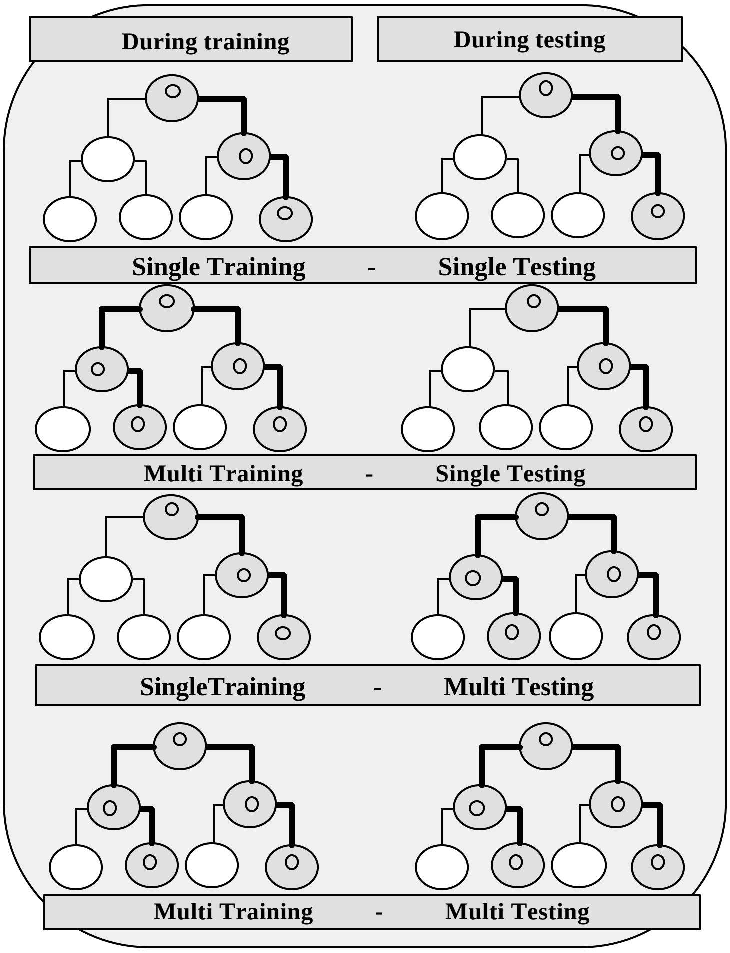

Figure 4: The paths a specific instance follows under different classes of solutions for the build/use of decision trees

6 Weight-Based Instance Replication

Another interesting extension to our basic algorithm is the balanced distribution of replicated instances. That is, rather than replicating instances every time they have to follow both branches of a sub-tree, we feed them to lower nodes using a weighting technique. When an instance is discretized, a number is attached to every discretized value of it. Those numbers indicate the weights with which initial values belong to disretized ones. Using the weights of the discretized values in conjunction with the splitting tests at each node we can calculate the total weight with which an instance arrives at a specific leaf. The sum of weights for every two newly created instances is equal with the weight of their father. This way, the sum of weights of instances at leaves is exactly the number of instances at root. The whole procedure does not affect the tree size or the paths from which instances traverse the tree. It only affects the majority scheme we use to decide the class of a testing instance.

We extended the basic algorithm with two different weighting techniques. The first one (*half-weighted*) assigns equal weights to the discretized values and thus, when it finds an instance that has to follow both branches of a node, it divides that instance's weight half to the right and half to the left subtree. Subsequent divisions of that instance result in further reduction of its weight over further nodes. The second technique (*weighted*) assigns weights to discretized values according to the distance of their initial value from the margins of the bucket that value belongs to.

Figure 5 depicts this procedure. An ellipse represents an instance and its area represents that instance's weight. Suppose that the buckets' line corresponds to a dscretized attribute (say B) and that a random instance has the value 2.5 at its B attribute. The original algorithm would discretize attribute B by giving it the values 1, 2 assuming that the weights for those values are both equal to one.

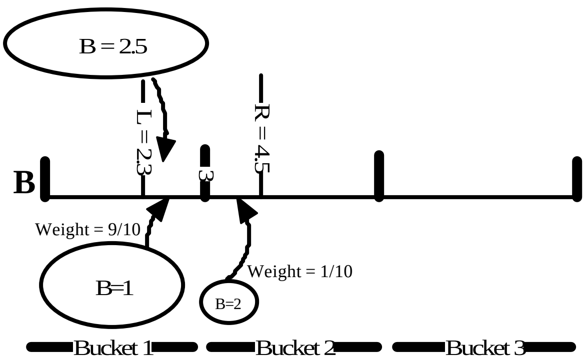

Figure 5: Discretization using the *weighted* algorithm

The *half-weighted* algorithm would also produce the values 1, 2 but with weights ½ and ½ for each one of them. Building on the same idea, the *weighted* algorithm computes the distance of 2.5 from the margins 2.3 and 4.5 and bears a higher weight for value 1 than for value 2 (as shown by the larger area for the discretized value *B*=1). That is because the value 2.5 is closer to 2.3 than to 4.5. To calculate the weight for the left discretized value we use the function: $\frac{R-x}{R-L}$ where R indicates the right and L the left margin (as in Figure 5); x indicates the value that is to be discretized. In the same manner, the weight for the right value can be computed using the function: $\frac{x-L}{R-L}$.

Now, suppose that a decision node has installed value 1 of attribute B as its test value. When the abovementioned instance arrives at that node it satisfies both branches and thus it has to follow both subtrees. On doing so, it uses the weights that derived at the discretization step (Figure 6).

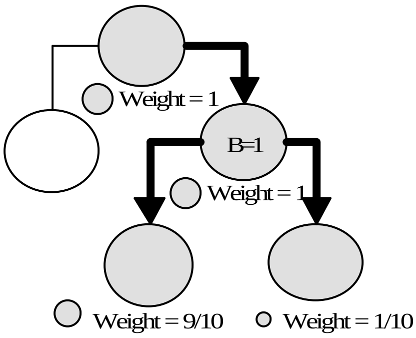

Figure 6: An instance's weights along the used paths

The comparison between the weighting techniques and our main algorithm is presented in Appendix E. It is clear that the weighted variant (although more promising at first glance) produces worse results than our basic algorithm. On the other hand, the half-weighted outcomes are comparable with the main algorithm. Still, they do nït justify the implementation overhead.

From a theoretical point of view, the proposed modifications absorb some of the set-values fuzziness by trying to reduce the effect of instance replication. Still, the introduced fuzziness is the very heart of the heuristic and the struggle to reduce (or control) it is somewhat premature at this point of the algorithm. This effect is more evident in the weighted variant, which minimizes fuzziness too soon and thus produces the worst results.

7 Discussion

We have proposed a modification to the classical decision tree induction approach in order to handle set-valued attributes with relatively straightforward conceptual extensions to the basic model. Our experiments have demonstrated that by employing a simple pre-processing stage, the proposed approach can handle efficiently numeric attributes (for a start) and yet yield significant accuracy improvements.

As we speculated in the introductory sections, the applicability of the approach to numeric attributes seemed all too obvious. Although numeric attributes have values that span a continuum, it is not difficult to imagine that conceptual groupings of similar objects would also share neighboring values. The instance space along any particular (numerical) axes usually demonstrates clusters of objects; these clusters are captured by the proposed discretization step. We view this as an example of everyday representation bias; we tend to favor descriptions that make it easier for us to group objects rather than allow fuzziness. This bias can be captured in a theoretical sense by the observation that entropy gain is maximized at class boundaries for numeric attributes [6].

When numeric attributes truly have values that make it difficult to identify class-related ranges, splitting will be "terminally" affected by the observed values and testing is affected too. Our approach serves to offer a second chance to near-misses and as the experiments show, near-misses do survive. The $\div^2$ approach to determine secondary bucket preference ensures that neighboring instances are allowed to be of value longer than the classical approach would allow. For the majority of cases this interaction has proved beneficial.

As far as the experiments are concerned, both *pruning level* and $\div^2$-*threshold* value have been proved important. There have been cases, where increasing $\div^2$-*threshold* decreased the accuracy (compared to lower $\div^2$-*threshold* values). We attribute such a behavior to the fact that the corresponding data sets do demonstrate a non-easily-separable class property. In such cases, increasing $\div^2$-*threshold*, thus making buckets more rigid, imposes an artificial clustering which deteriorates accuracy. That our approach still over-performs un-pruned ITI is a sign that the few instances which are directed to more than one bucket compensate for the non-flexibility of the "few" buckets available.

The proposed algorithm can also find a potential application area wherever instances with symbolic set-valued attributes are the case. Natural language processing serves as an example. Orphanos *et al.* [14] exemplified the use of decision trees on Part-Of-Speech tagging for Modern Greek, a natural language which, from the computational perspective, has not been widely investigated. Orphanos *et al.* used three different algorithms (included the proposed one) that were able to handle symbolic set-valued attributes, a requirement posed by the nature of the linguistic datasets. The proposed algorithm outperformed the other two by a comfortable margin, demonstrating the value of pre-pruning too.

Scheduled improvements to the proposed approach for handling set-valued attributes span a broad spectrum. It is interesting to note that using set-valued attributes naturally paves the way for deploying more effective post-processing schemes for various tasks (character recognition is an obvious example). By following a few branches, one need not make a guess about the second best option based on only one node, as is usually the case. Our coverage results show that significant gains can be materialized.

Another important line of research concerns the development of a suitable post-pruning strategy. Instance replication complicates the maths involved in estimating error, and standard approaches need to be better studied (for example, it is not at all obvious how minimum-description-length pruning might be readily adapted to set-valued attributes). Furthermore, we need to explore the extent to which instance replication does (or, hopefully, does not) seriously affect the efficiency of incremental decision tree algorithms.

Recall that the algorithm depends on various parameters ($\div^2$, *max-buckets, pruning-level*) that often need special tuning -over a specific database- so as to obtain optimal results regarding accuracy and size. It seems worthy to build on a meta-learner scheme where the algorithm would self-decide (via experiments) the values for its parameters. We are envisioning such a scheme that uses genetic algorithms to evolve the parameters.

We feel that the proposed approach opens up a range of possibilities for decision tree induction. By beefing it up with the appropriate mathematical framework we may be able to say that a little bit of carefully introduced fuzziness improves the learning process.

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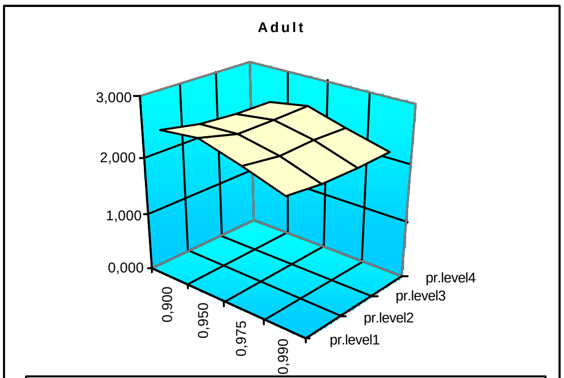
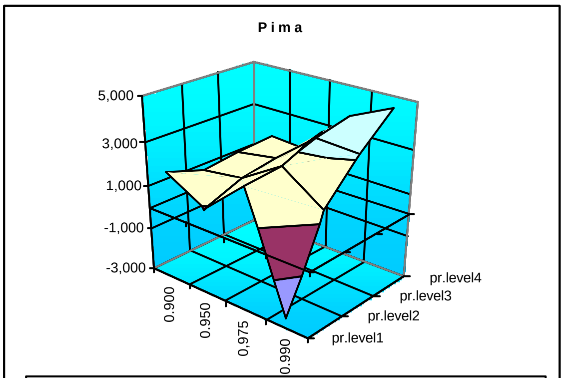
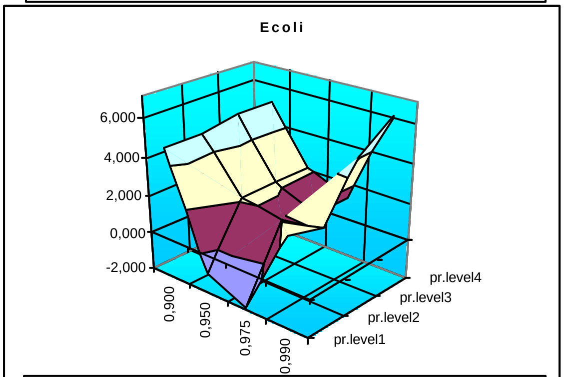
Appendix A_1: Accuracy Difference
Multi-Training/Multi-Testing *against* ITI *(unpruned trees)*
(Results above zero indicate a better accuracy for Multi-Training/Multi-Testing)
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference



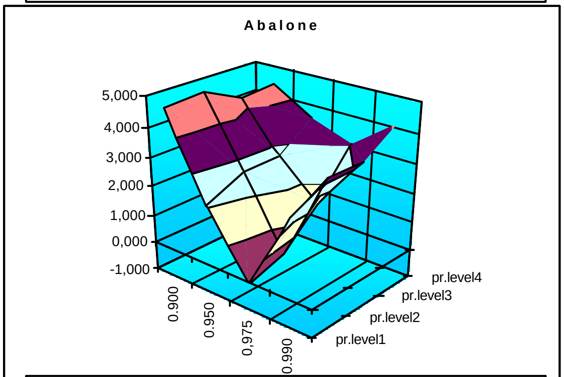
ITI: **79.70 unpruned**



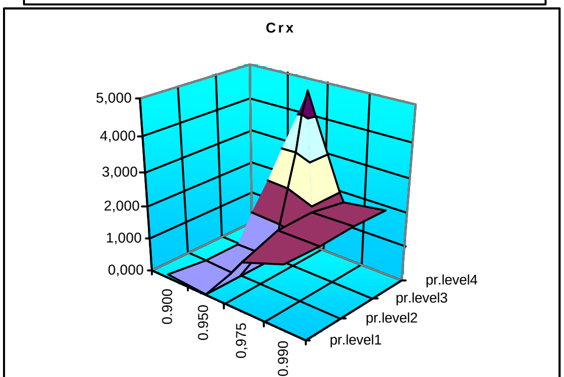
ITI: **68.29 unpruned**



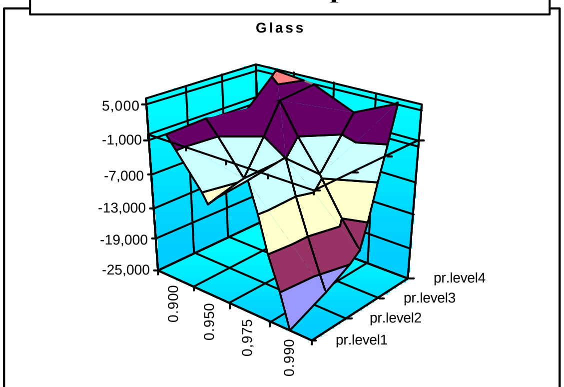
ITI: **65.76 unpruned**



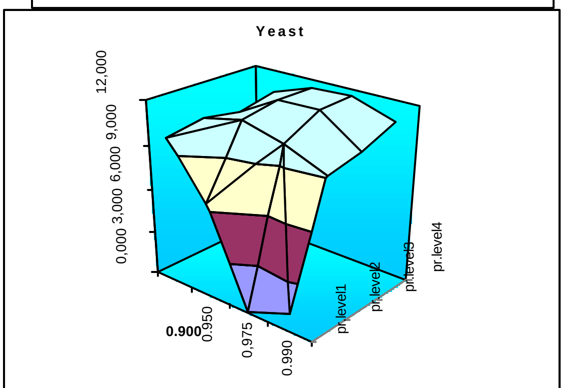
ITI: **48.03 unpruned**



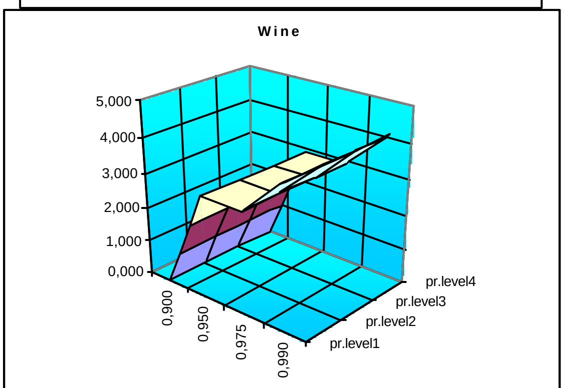
ITI: **78.17 unpruned**



ITI: **38.57 unpruned**



ITI: **43.92 unpruned**

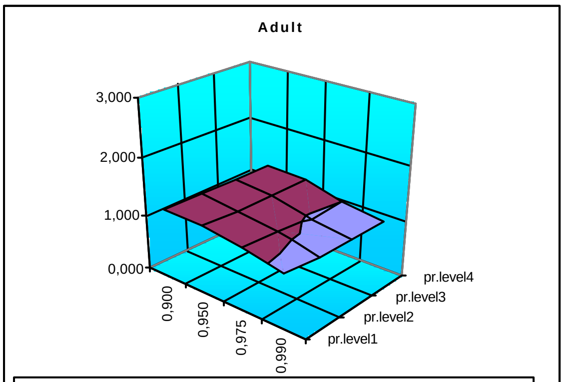


ITI: **87.06 unpruned**

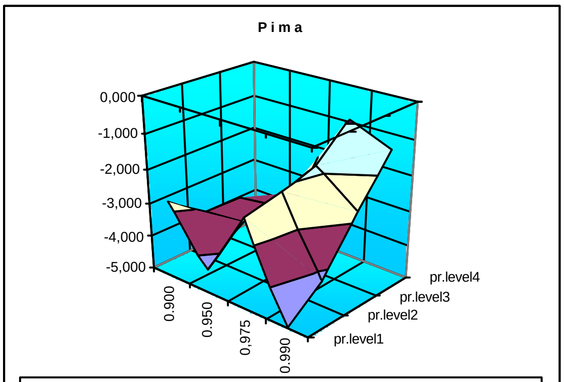
Appendix A_2: Accuracy Difference
Multi-Training/Multi-Testing *against* ITI *(pruned trees)*

(Results above zero indicate a better accuracy for Multi-Training/Multi-Testing)

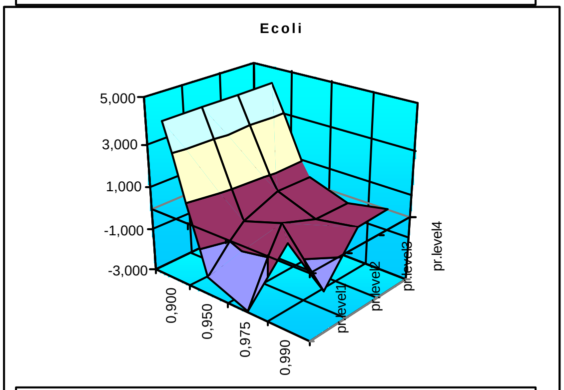
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference



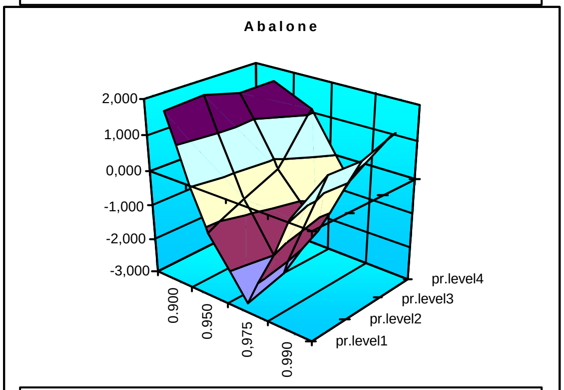
ITI: **82.42 pruned**



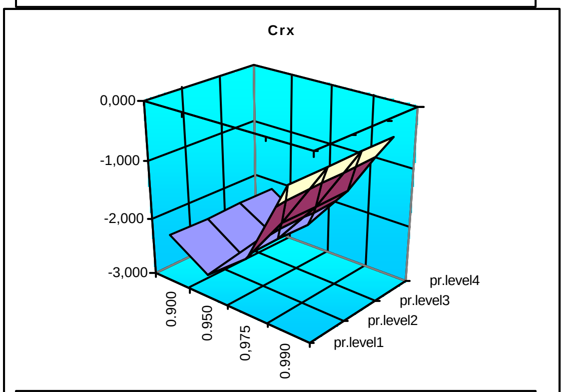
ITI: **73.68 pruned**



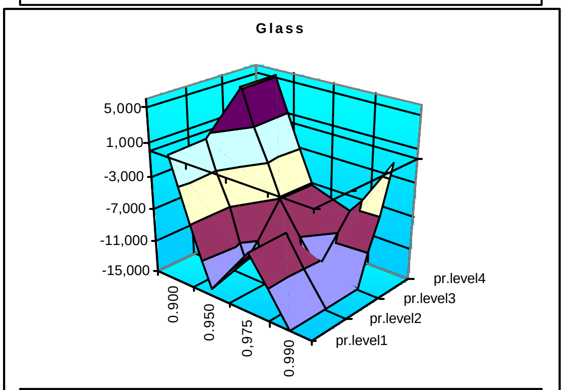
ITI: **66.37 pruned**



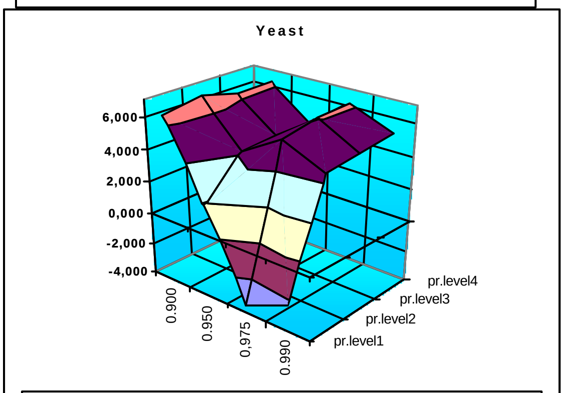
ITI: **50.60 pruned**



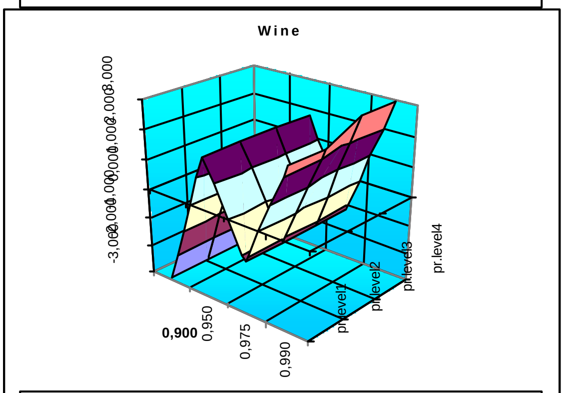
ITI: **83.33 pruned**



ITI: **36.67 pruned**

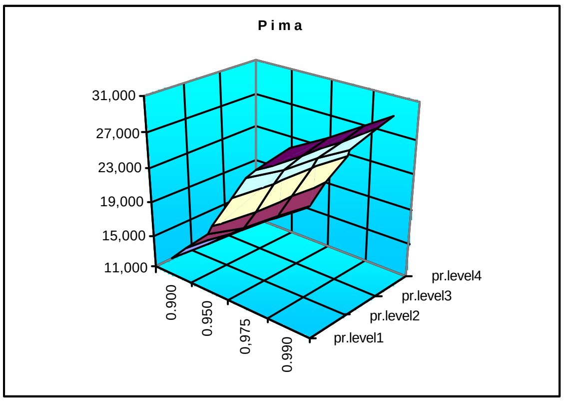


ITI: **47.70 pruned**

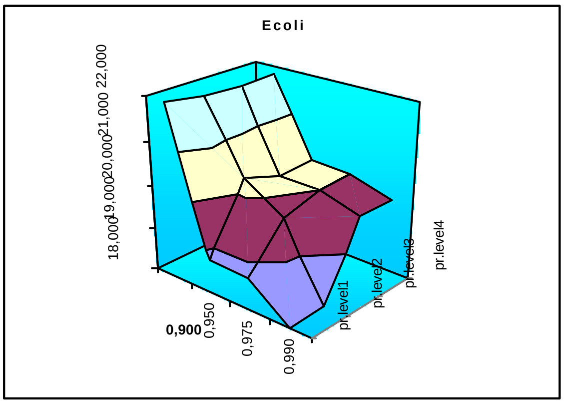


ITI: **88.24 pruned**

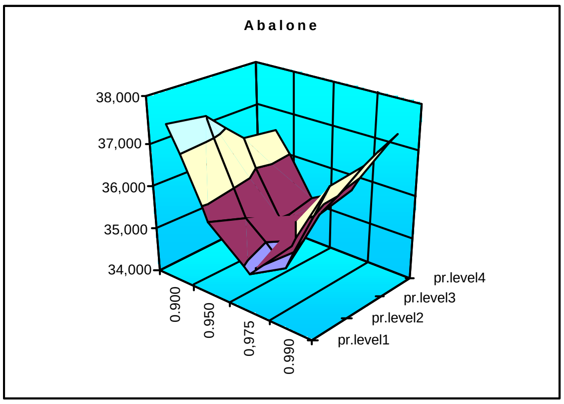
Appendix B_1: Accuracy Difference for Coverage2
Multi-Training/Multi-Testing *against* ITI *(unpruned trees)*
(Results above zero indicate a better accuracy for Multi-Training/Multi-Testing)
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference



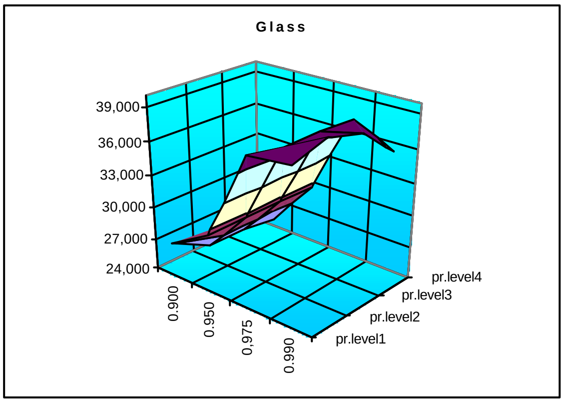
ITI: **79.70 unpruned**



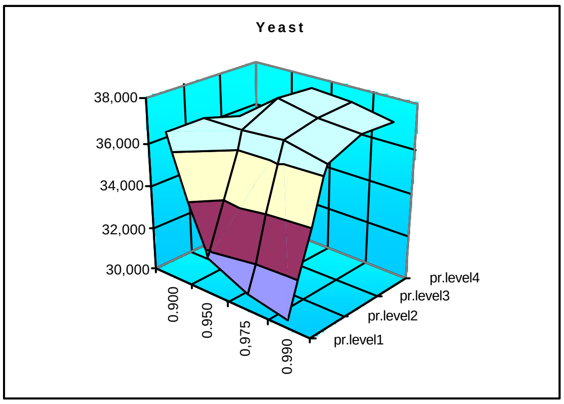
ITI: **65.76 unpruned**



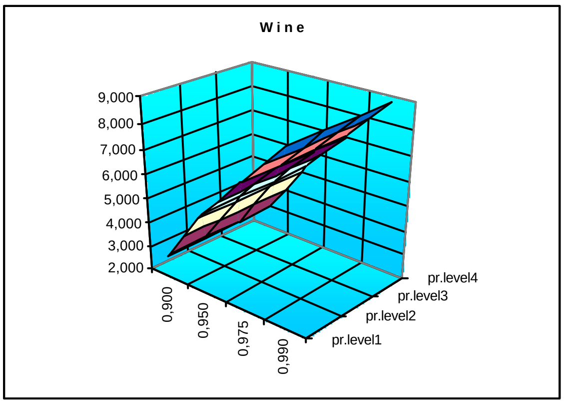
ITI: **48.03 unpruned**



ITI: **43.92 unpruned**

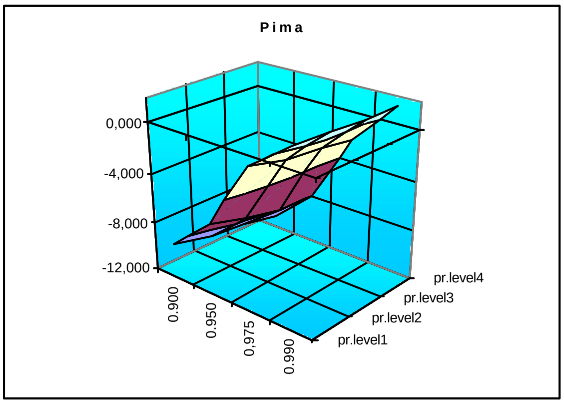


ITI: **87.06 unpruned**

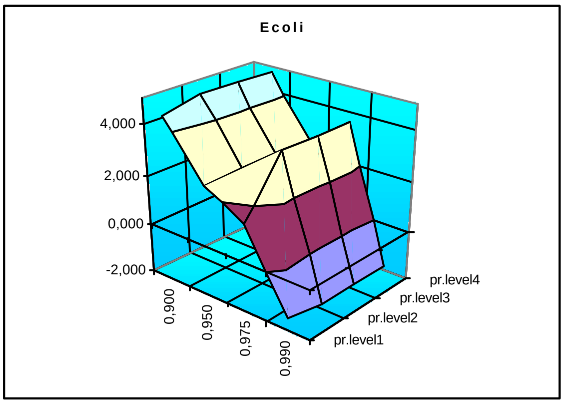


ITI: 38.57 **unpruned**

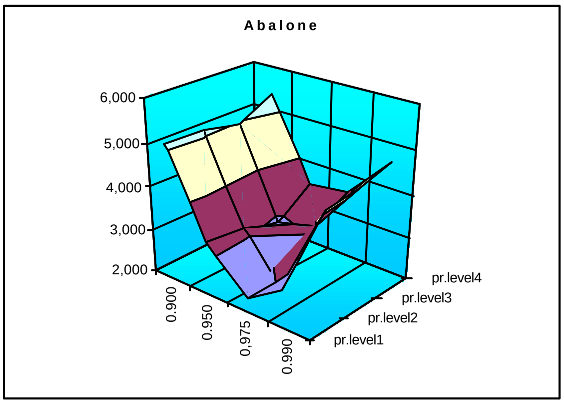
Appendix B_2: Accuracy difference for Coverage2
Multi-Training/Multi-Testing *against* ITI *(pruned trees)*
(Results above zero indicate a better accuracy for Multi-Training/Multi-Testing)
x-axis: x-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference



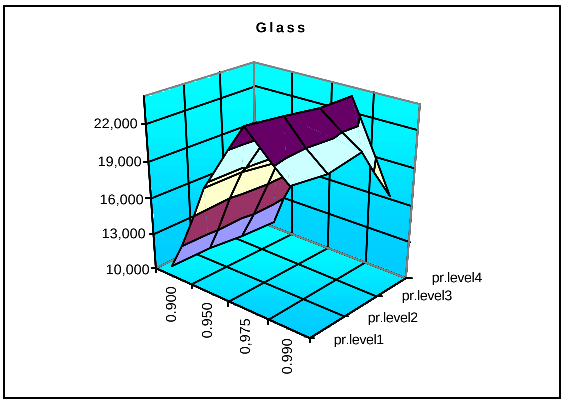
ITI: 97.50 pruned



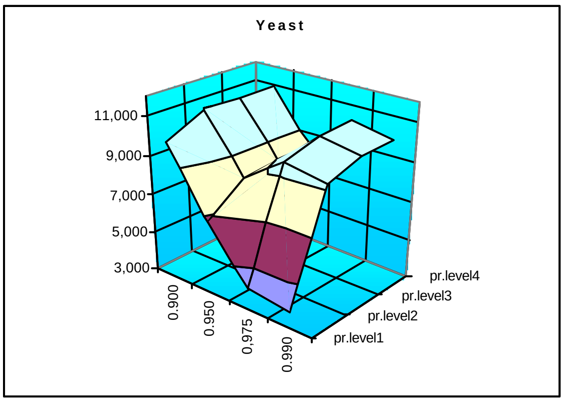
ITI: 81.51 pruned



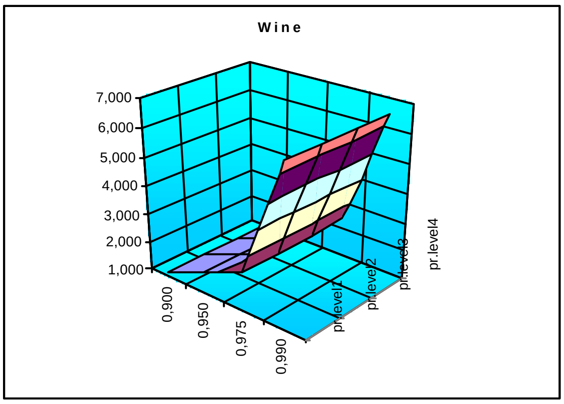
ITI: 80.86 pruned



ITI: 52.86 pruned

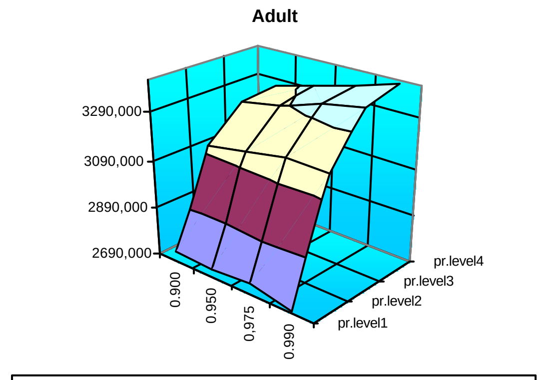


ITI: 70.07 pruned

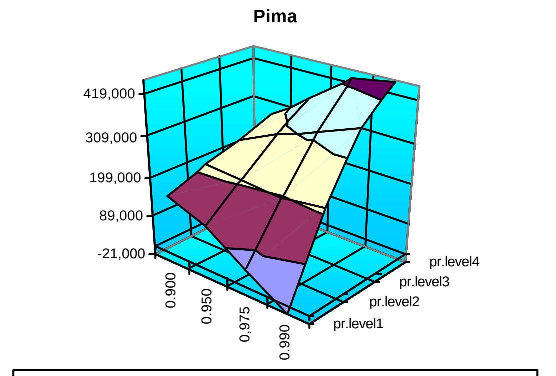


ITI: 92.94 pruned

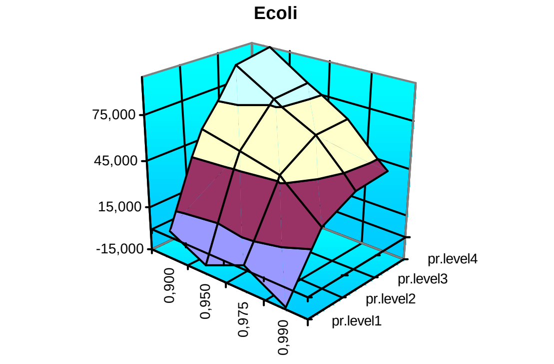
Appendix C_1: Size difference
Multi-Training/Multi-Testing *against* ITI
(Results above zero indicate a better size for Multi-Training/Multi-Testing)
x-axis: x-square threshold, **y-axis**: pruning level, **z-axis**: size difference



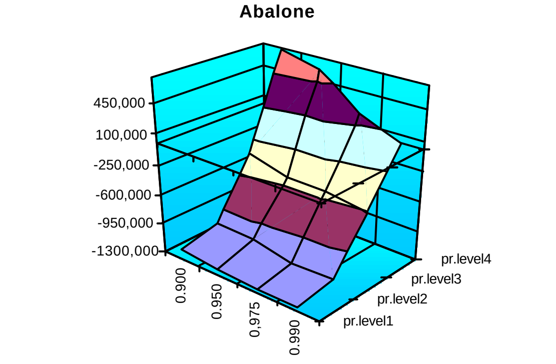
ITI: 3331,667 unpruned



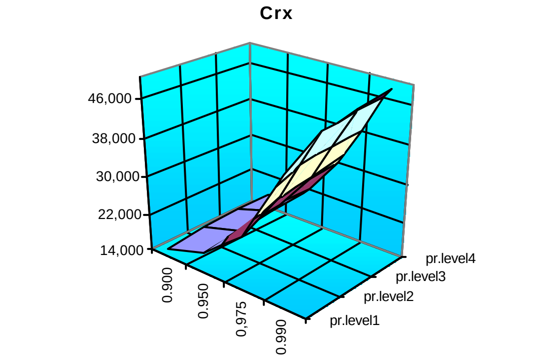
ITI: 134 unpruned



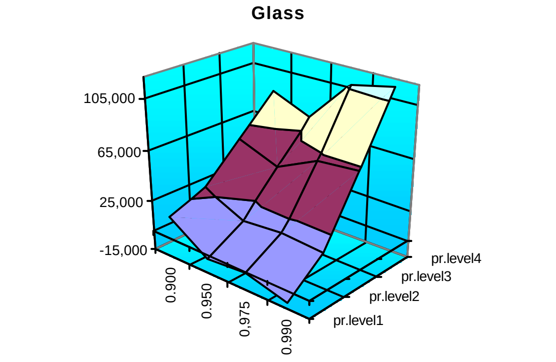
ITI: 69,667 unpruned



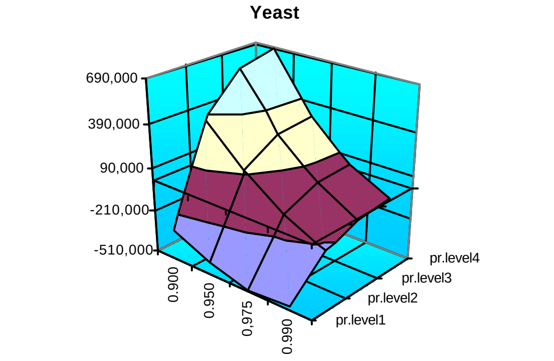
ITI: 1306 unpruned



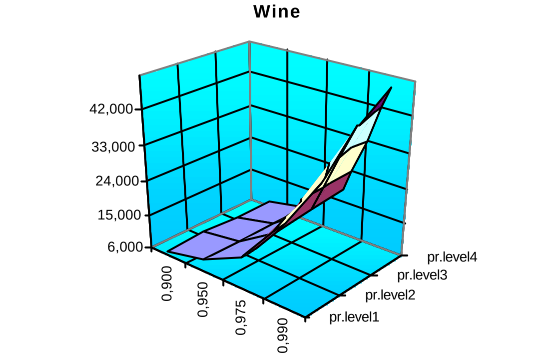
ITI: 95 unpruned



ITI: 47,667 unpruned

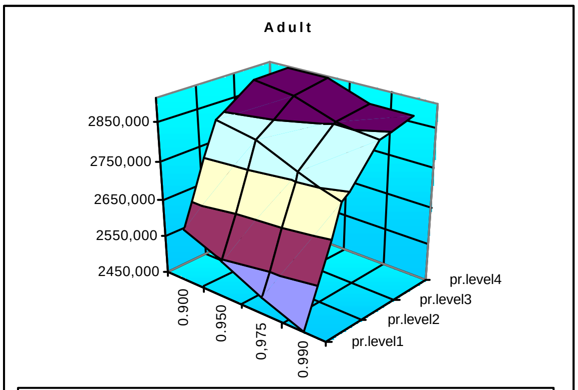


ITI: 572,667 unpruned

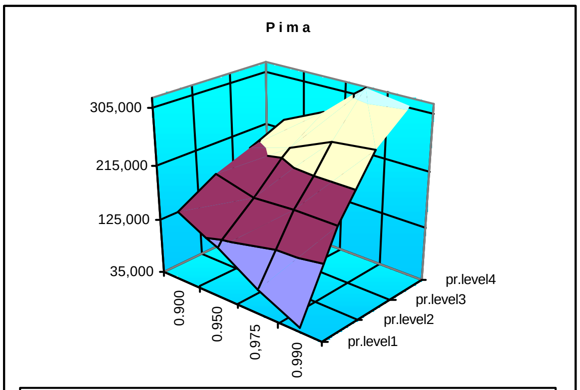


ITI: 10,6 unpruned

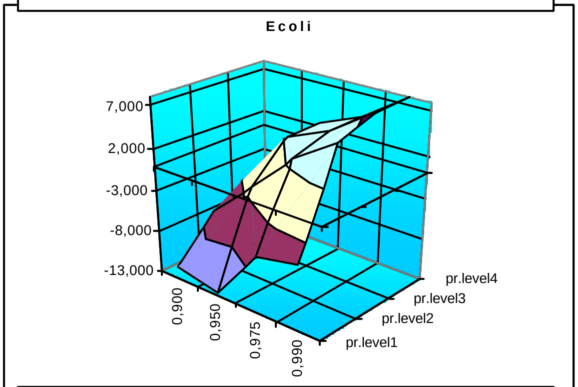
Appendix C_2: Size difference
Multi-Training/Multi-Testing *against* ITI
(Results above zero indicate a better size for Multi-Training/Multi-Testing)
x-axis: x-square threshold, **y-axis**: pruning level, **z-axis**: size difference



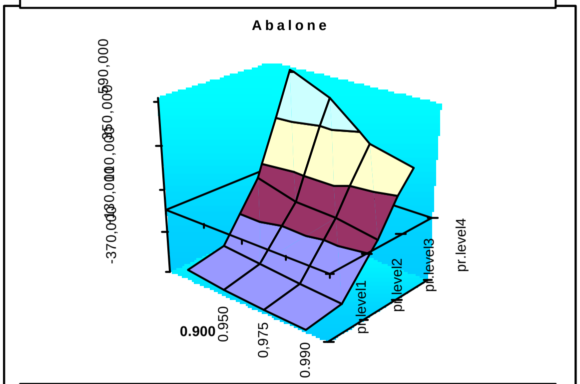
ITI: 665 pruned



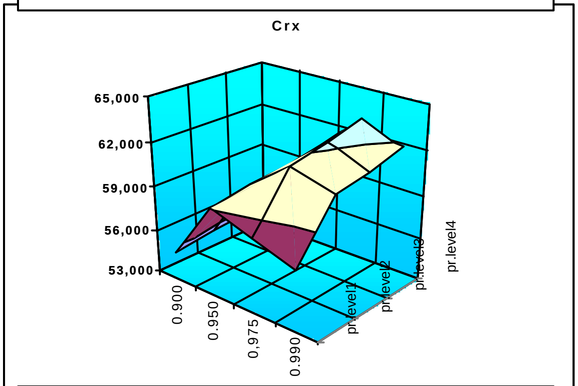
ITI: 32 pruned



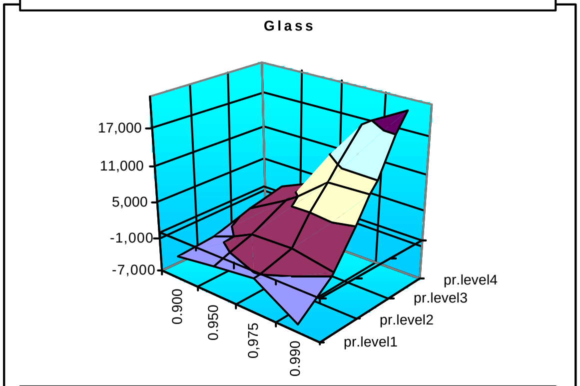
ITI: 26,67 pruned



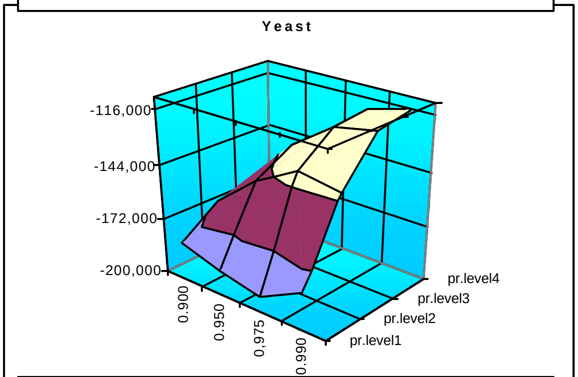
ITI: 371,67 pruned



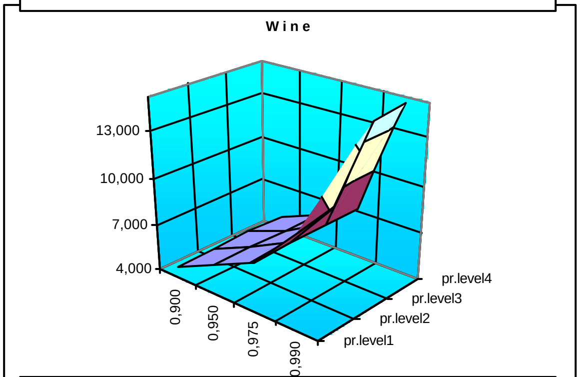
ITI: 16,33 pruned



ITI: 22,333 pruned



ITI: 202,67 pruned

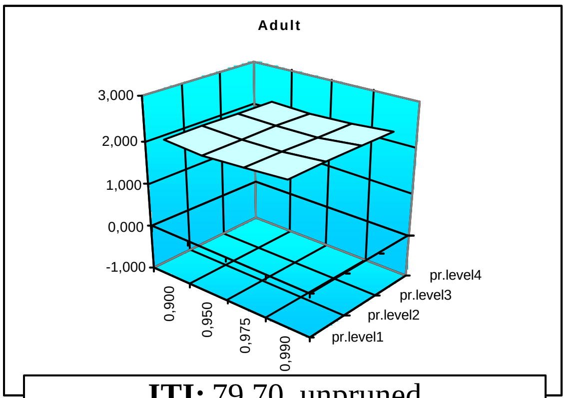


ITI: 5,8 pruned

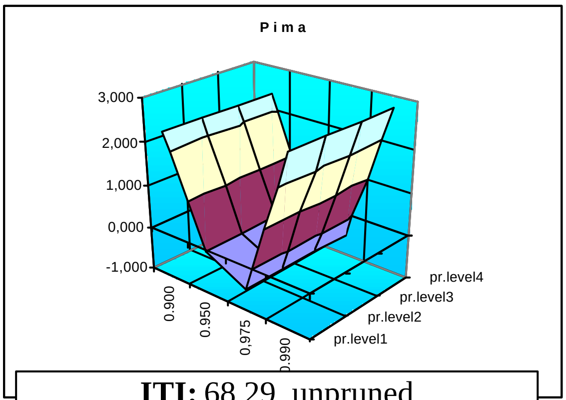
Appendix D_1: Accuracy Difference
Single-Training/Single-Testing *against* ITI

(Results above zero indicate better accuracy for Single-Training/Single-Testing)

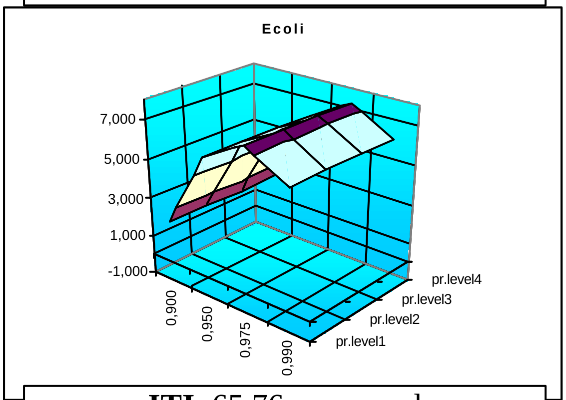
x-axis: x-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference



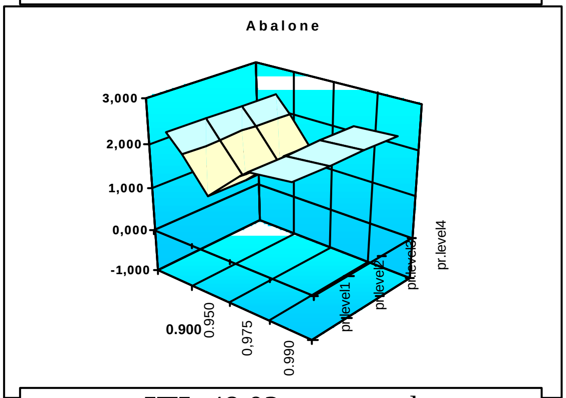
ITI: 79.70 unpruned



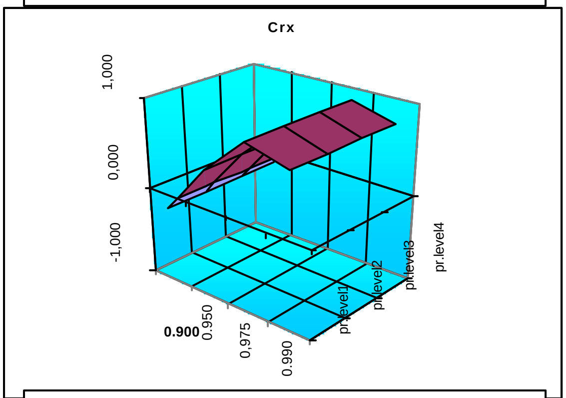
ITI: 68.29 unpruned



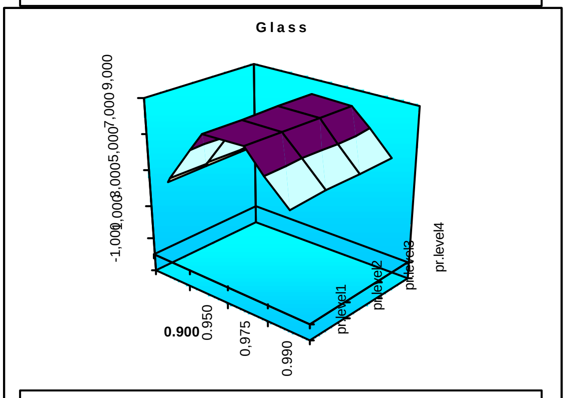
ITI: 65.76 unpruned



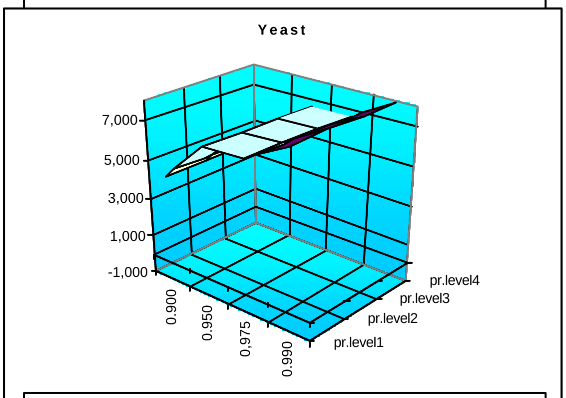
ITI: 48.03 unpruned



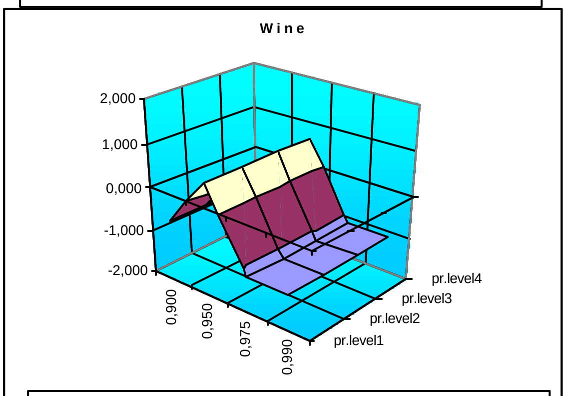
ITI: 78.17 unpruned



ITI: 38.57 unpruned



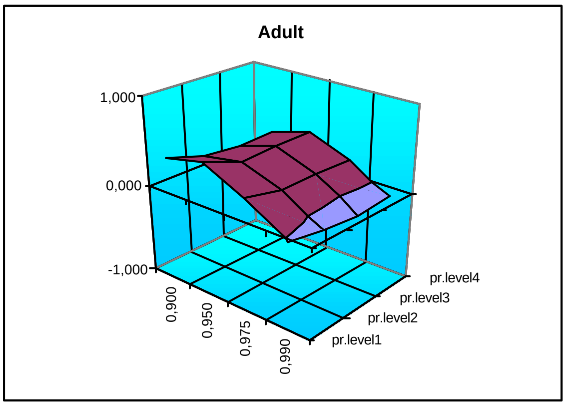
ITI: 43.92 unpruned

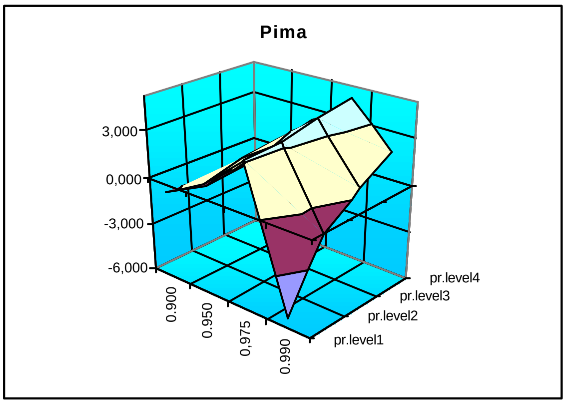


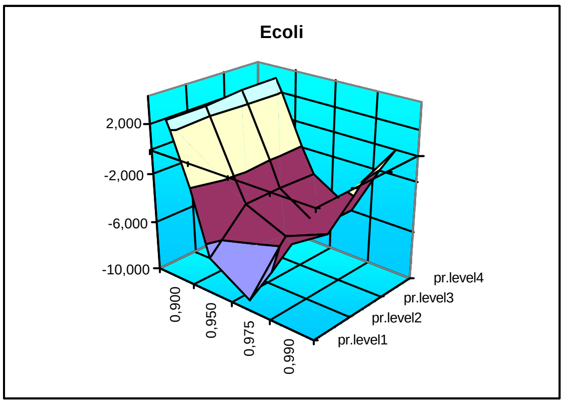
ITI: 87.06 unpruned

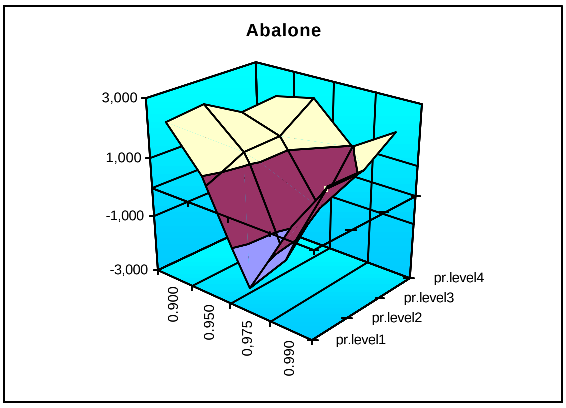
Appendix D_2: Accuracy Difference
Multi-Training/Multi-Testing *against* Single-Training/Single-Testing
(Results above zero indicate better accuracy for Multi-Training/Multi-Testing)

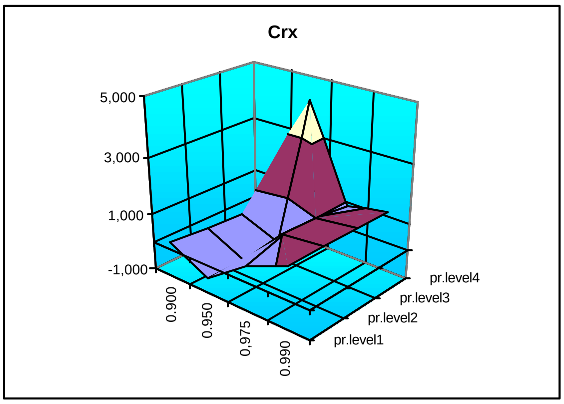
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference

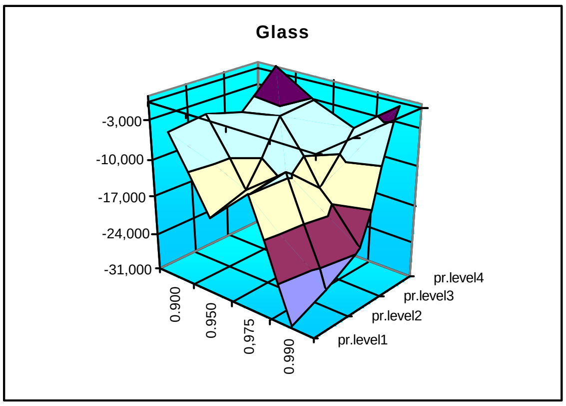


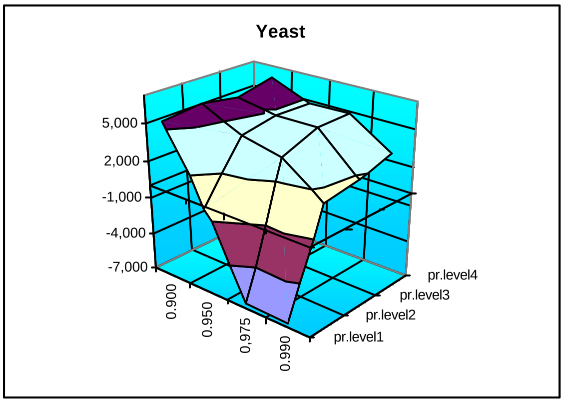


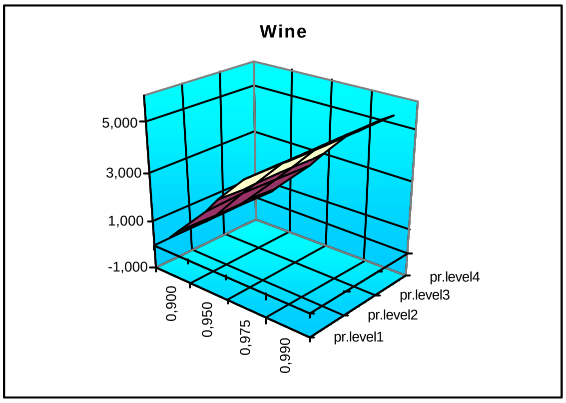




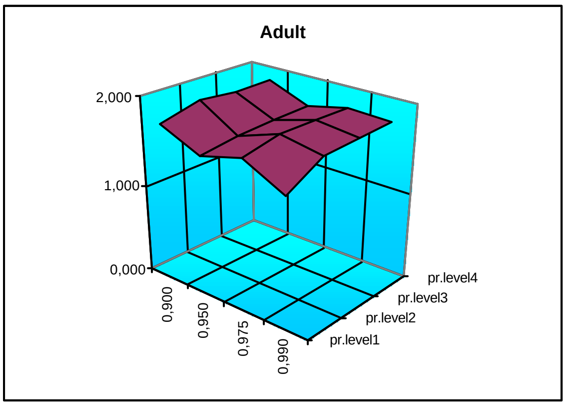


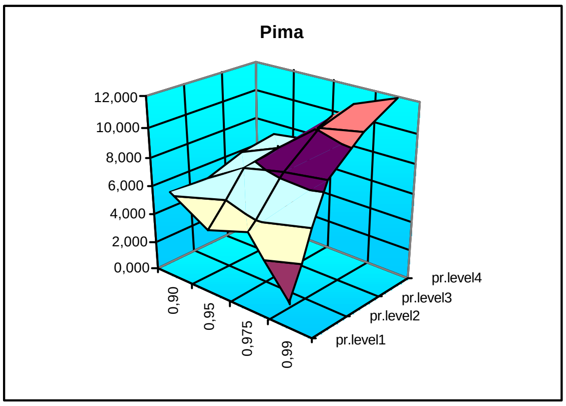


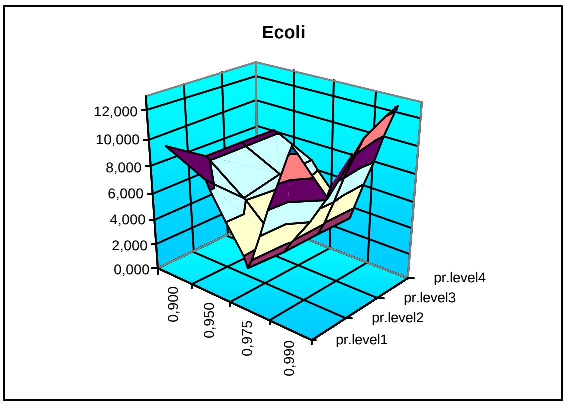


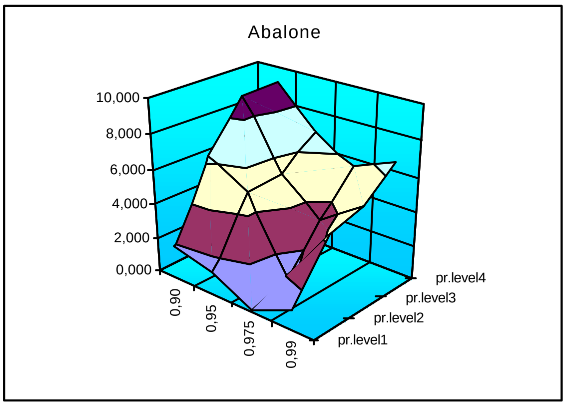


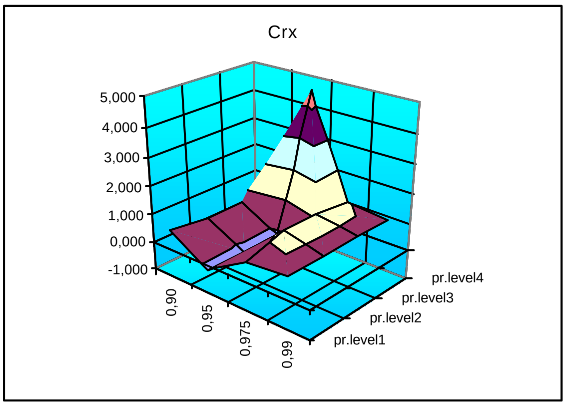
Appendix D_3: Accuracy Difference
Multi-Training/Multi-Testing *against* Multi-Training/Single-Testing
(Results above zero indicate better accuracy for Multi-Training/Multi-Testing)
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference

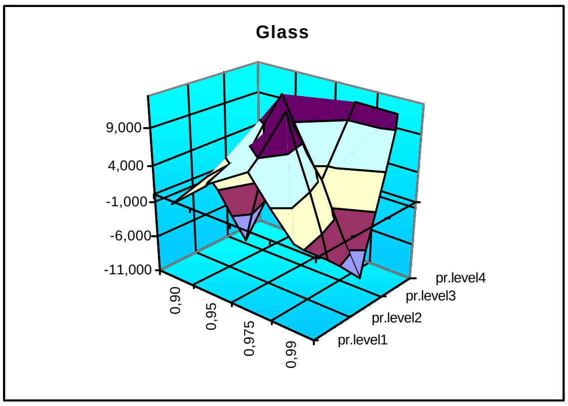


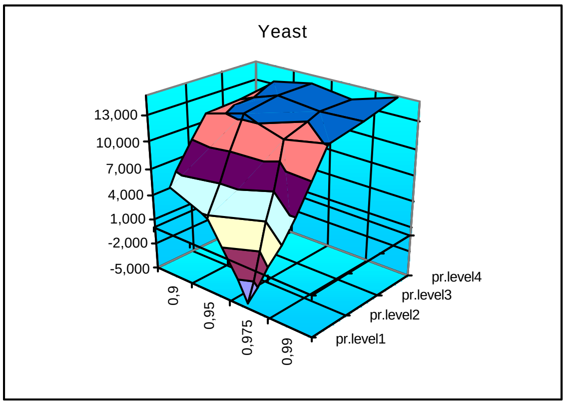


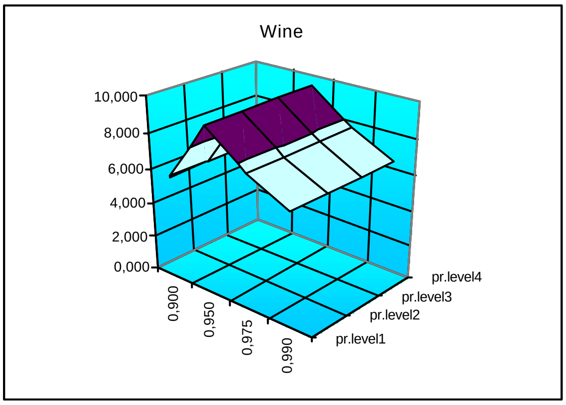




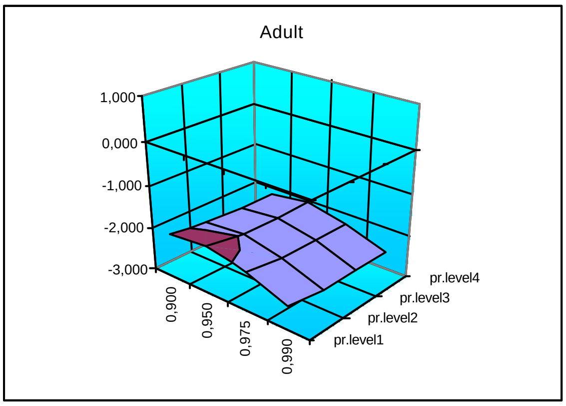


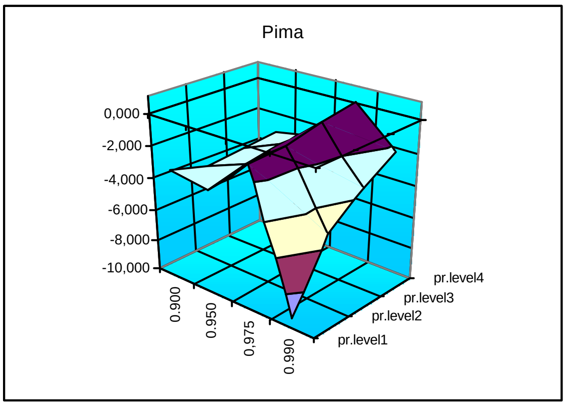


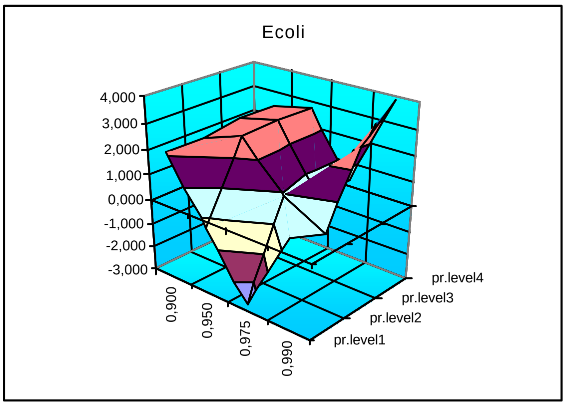


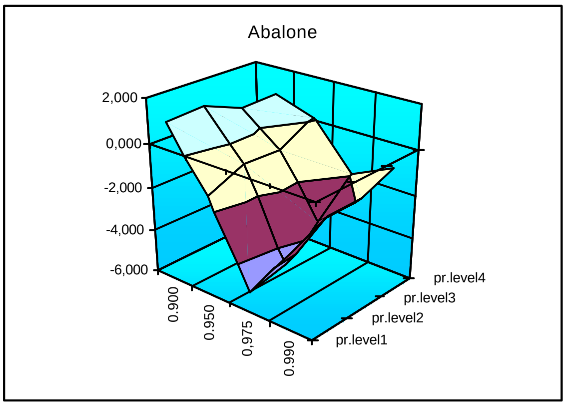


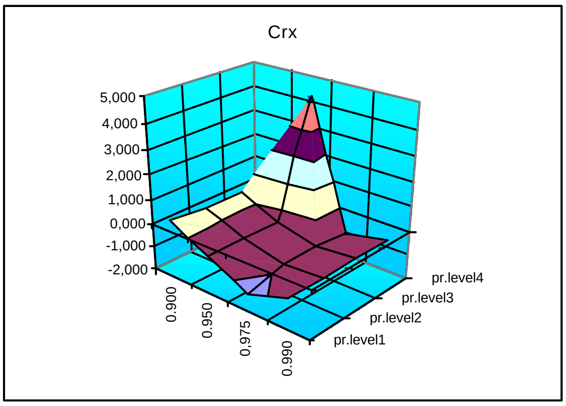
Appendix D_4: Accuracy Difference
Multi-Training/Multi-Testing *against* Single-Training/Multi-Testing
(Results above zero indicate better accuracy for Multi-Training/Multi-Testing)
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference

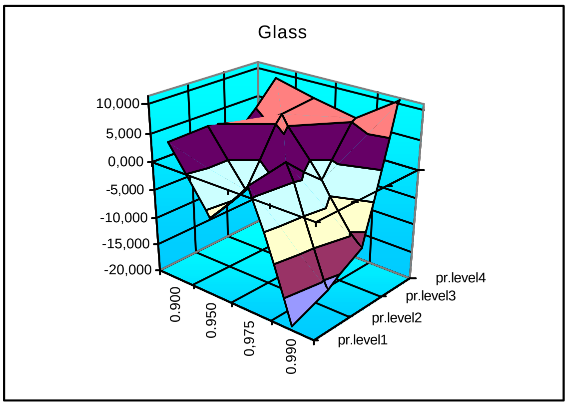


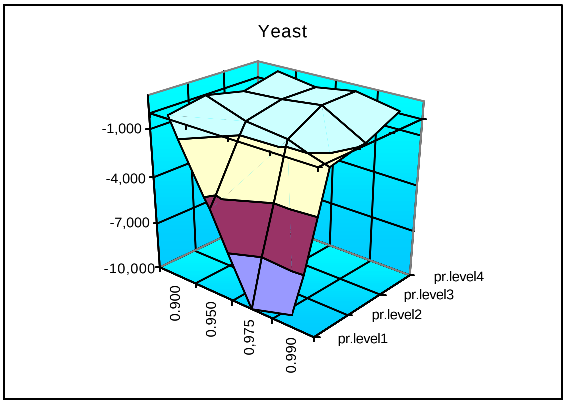


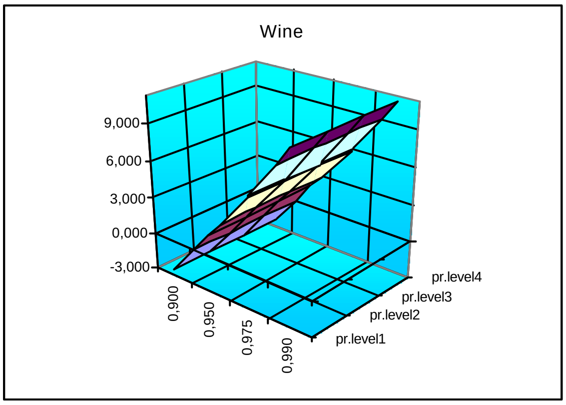










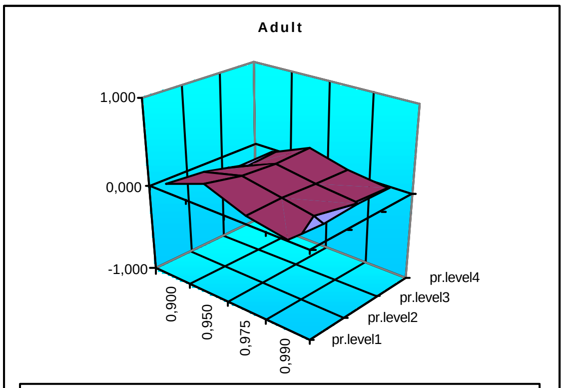


Appendix E_1: Accuracy Difference

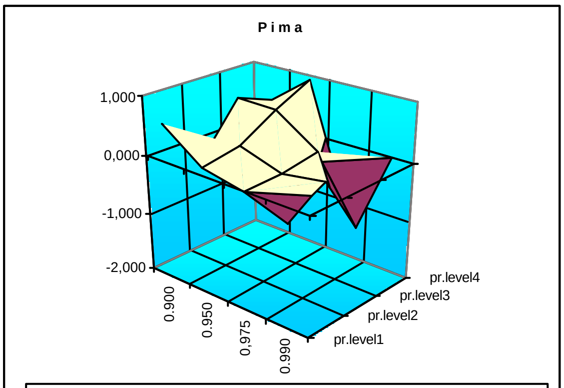
Multi-Training/Multi-Testing *against* Multi-Training/Multi-Testing (*Half-Weighted*)

(Results above zero indicate better accuracy for Multi-Training/Multi-Testing)

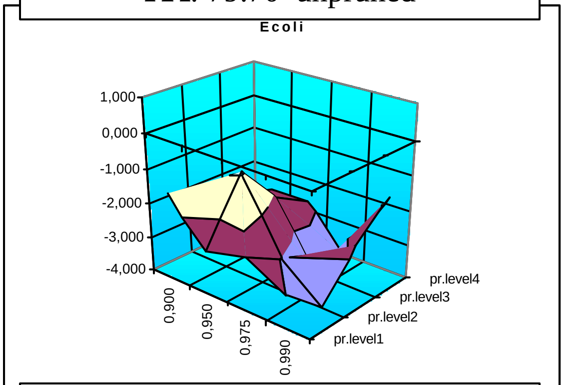
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis** : accuracy difference



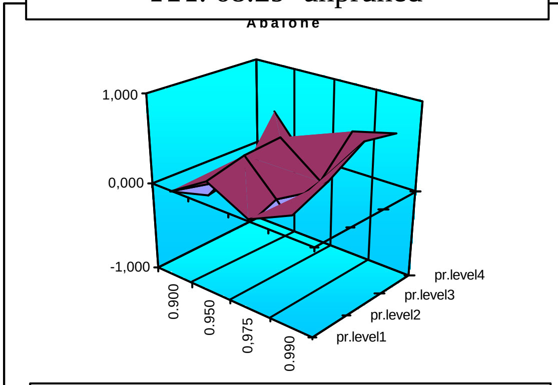
ITI: 79.70 unpruned



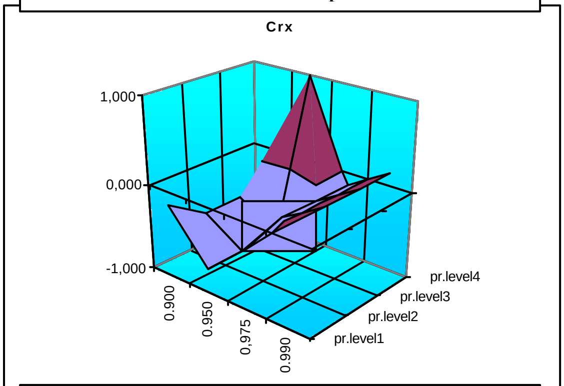
ITI: 68.29 unpruned



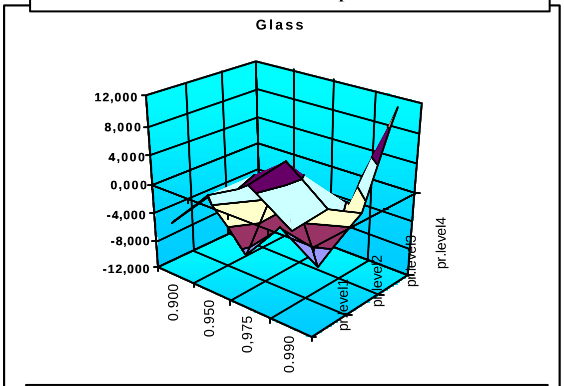
ITI: 65.76 unpruned



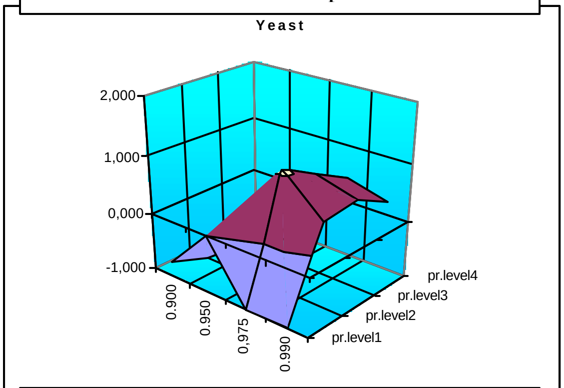
ITI: 48.03 unpruned



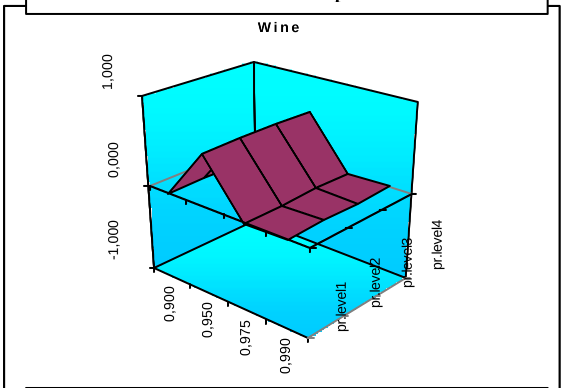
ITI: 78.17 unpruned



ITI: 38.57 unpruned



ITI: 43.92 unpruned

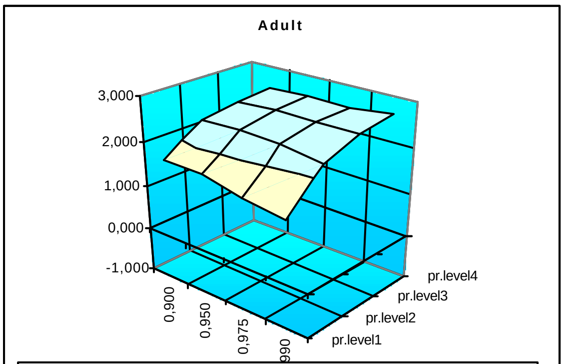


ITI: 87.06 unpruned

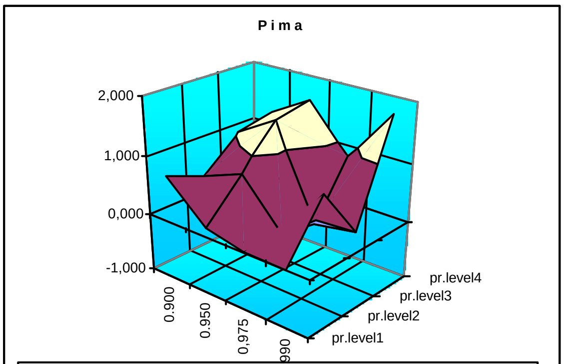
Appendix E_2: Accuracy Difference
Multi-Training/Multi-Testing *against* Multi-Training/Multi-Testing (*Weighted*)

(Results above zero indicate better accuracy for Multi-Training/Multi-Testing)

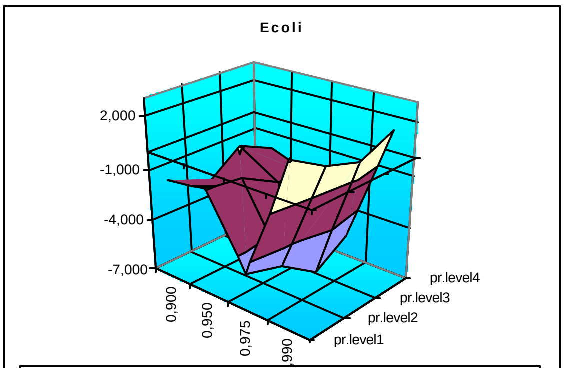
x-axis: a-square threshold, **y-axis**: pruning level, **z-axis**: accuracy difference



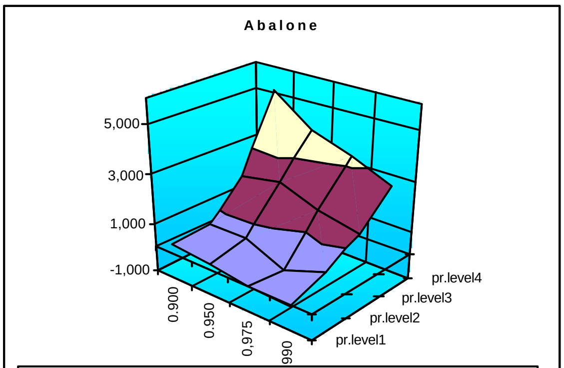
ITI: 79.70 unpruned



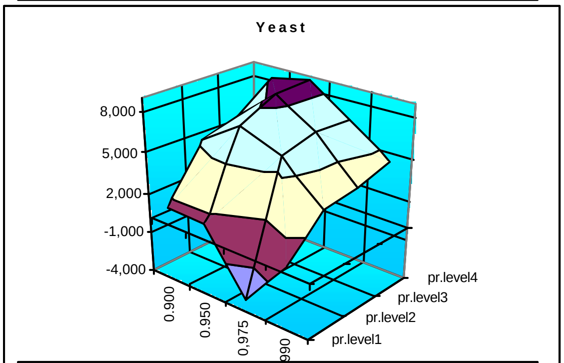
ITI: 68.29 unpruned



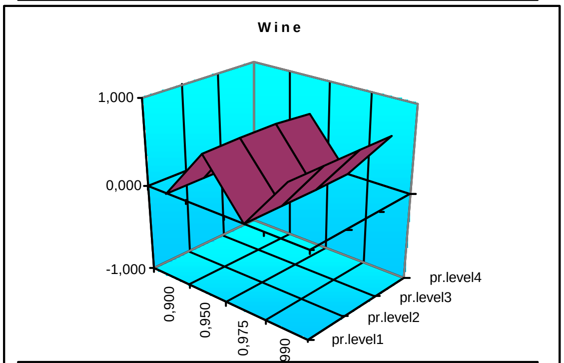
ITI: 65.76 unpruned



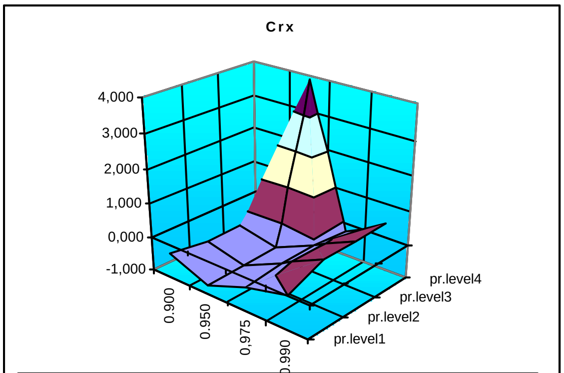
ITI: 48.03 unpruned



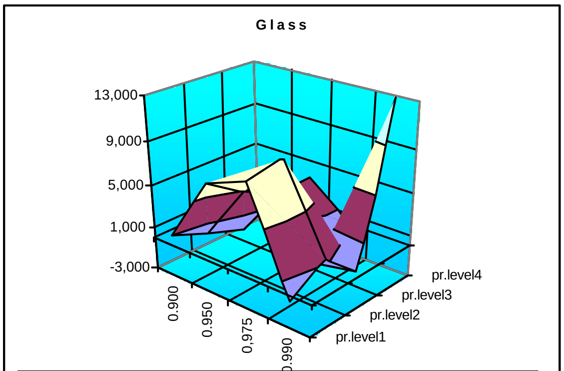
ITI: 78.17 unpruned



ITI: 38.57 unpruned



ITI: 43.92 unpruned



ITI: 87.06 unpruned