Tracing Cluster Transitions for Different Cluster Types

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Abstract. Clustering algorithms detect groups of similar population members, like customers, news or genes. In many clustering applications the observed population evolves and changes, subject to internal and external factors. Detecting and understanding change is important for decision support. We extend our earlier framework MONIC for generic cluster transition modeling and detection, into MONIC$^+$ for cluster-type specific transition detection. MONIC$^+$ encompasses a typification of clusters and cluster-type-specific transition indicators, by exploiting cluster topology and cluster statistics for transition detection.

1 Introduction

For many clustering applications, clusters should not be observed as static objects, since the underlying datasets undergo changes over time, e.g. customers and their buying preferences, scientific publications and their topics or viruses and their resistance to medicaments. Research on spatiotemporal clustering, incremental clustering and stream clustering addresses the problem by adapting clusters to changing datasets. However, the *tracing and understanding* of the changes themselves is of no less importance for effective decision support.

In our previous work [10], we proposed the MONIC framework for cluster transition detection. MONIC is independent of the clustering algorithm since it relies on the contents of the underlying data stream. However, due to its generality, MONIC does not exploit the particular features of the different cluster types for transition detection. We extend here MONIC into MONIC$^+$ that covers the special characteristics associated with different cluster types, thus allowing us to capture cluster-type-specific transitions.

After discussing related work in Section 2, we introduce in Section 3 a typification of clusters and then specify the notion of match for clusters derived at different timepoints over an accumulating data stream. Section 4 contains our cluster transition monitoring method and heuristics for different cluster types. In Section 5 we present our first experiments. Section 6 concludes our study.

2 Related Work

Research relevant to our work can be categorized into methods for cluster change detection and methods for spatiotemporal clustering. Among the former, the *FO-*

CUS framework [6] compares two datasets and computes their deviation based on the data mining models they induce. Clusters are a special case of models represented as non-overlapping regions that are described through a set of attributes and correspond to a set of raw data. However, the emphasis in this work is on comparing datasets, not in understanding how a cluster has evolved inside a new clustering. The *PANDA framework* [4] proposes methods for the comparison of simple patterns and aggregation logics for the comparison of complex ones. PANDA concentrates on the generic and efficient realization of pattern comparison, rather than on the detection and interpretation of cluster transitions. In [1] clusters are modeled as kernel functions and changes as *kernel density changes* at each spatial location; different types of change are considered, with emphasis in computing change velocity. In [13], *formation and dissipation events* are detected upon clusters of spatial scientific data. Both approaches operate upon a specific attribute space, the 2D spatial space; they observe a cluster as a densification in the time-invariant feature space and monitor changes upon it. Hence, these methods cannot be coupled with arbitrary clustering algorithms, e.g. hierarchical algorithms, density based algorithms or even clustering algorithm over dynamic attribute spaces. Moreover, these methods juxtapose each cluster to the feature space and cannot trace interferences among clusters, e.g. one cluster absorbing the other. In [9] a special type of change is proposed, the *moving cluster*, which traces common data records between clusters of consecutive timepoints. Our work, however, is more general since it encompasses several cluster transition types.

3 A Model for Clusters over Dynamic Data

We assume that data are clustered at timepoints $t_1, \ldots, t_n$. Clustering ζ_i, derived at timepoint t_i, corresponds to a partitioning of the dataset D_i seen thus far. As is typical in data streams, we allow for the decay of old records: We use an *ageing function* $age(x, t_i) \in [0, 1]$ that assigns a weight to each record x seen at t_i or earlier, so as the most recent records are assigned higher weights. The simplest form of this function is a sliding window.

Our goal is to trace/monitor a cluster found at some timepoint among the clusters of the next timepoint. Since this depends on the notion of cluster itself, we first introduce a typification of clusters, which we use next to derive type-specific concepts for the comparison of clusters across the time axis.

3.1 Typification of Cluster Definitions

Clustering algorithms use a variety of cluster definitions [8]. We propose the following typification that facilitates the study of clusters as *changing objects*:
[Type A] Clusters are discovered upon a *dataset-independent* metric space. A cluster is a geometric object, e.g. a sphere like in K-means. Cluster changes are observed over the static metric space as geometric transformations.

[Type B1] There is no metric space or it depends on the contents of the dataset at each timepoint. A cluster is defined *extensionally* as a set of data records. Hierarchical algorithms which build dendrograms and express clusters as sets of proximal data points belong to this type. These algorithms use a metric space to derive a clustering on a dataset, but this space is *data-dependent*, in the sense that the addition of a new record might change the border of a cluster, even if this record does not belong to the cluster at all.
[Type B2] A cluster is defined *intensionally* as a distribution. For a cluster X of type B2, we denote its cardinality as $card(X)$, its mean as $\mu(X)$ and its standard deviation as $\sigma(X)$. The Expectation-Maximization (EM) algorithm belongs to this category. Several combinations of the base types are possible, e.g. when both the dataset and its statistics are used (types B1+B2).

3.2 Cluster Matching

A *cluster transition* is a change effected upon a cluster $X \in \zeta_i$ discovered at t_i, when we observe it at the next timepoint t_j. The first step in detecting a transition is the tracing of X in the clustering ζ_j of t_j – if it still exists. We define the notion of *overlap* and of (best) *match* for a cluster, before we proceed with a categorization of cluster transitions.

Definition 1 (Cluster overlap). *Let ζ_i be the clustering discovered at timepoint t_i and ζ_j the one discovered at $t_j, j \neq i$. We define a function overlap() that computes the similarity or overlap of a cluster $X \in \zeta_i$ towards a cluster $Y \in \zeta_j$ as a value in $[0, 1]$ such that (i) the value 1 indicates maximum overlap, while 0 stands for no overlap and (ii) it holds that $\sum_{Y \in \zeta_j} overlap(X, Y) \leq 1$.*

Cluster overlap is defined asymmetrically. After this generic definition of the overlap function, we specify *overlap*() for each cluster type.

Definition 2 (Overlap for Type A Clusters). *Let ζ_i, ζ_j be two clusterings of Type A clusters, derived at $t_i < t_j$ respectively. For two clusters $X \in \zeta_i$ and $Y \in \zeta_j$, the overlap of X to Y is the normalized intersection of their areas:*

$$overlap(X, Y) = \frac{area(X) \cap area(Y)}{area(X)}$$

Definition 3 (Overlap for Type B1 Clusters). *Let ζ_i, ζ_j be two clusterings of Type B1 clusters, derived at $t_i < t_j$ respectively. For two clusters $X \in \zeta_i$ and $Y \in \zeta_j$, the overlap of X to Y equals to the normalized sum of the weights of their common data points:*

$$overlap(X, Y) = \frac{\sum_{a \in X \cap Y} age(a, t_j)}{\sum_{x \in X} age(x, t_j)}$$

Definition 4 (Overlap for Type B2 Clusters). *Let ζ_i, ζ_j be two clusterings of Type B2 clusters, derived at $t_i < t_j$ respectively. For two clusters $X \in \zeta_i$ and*

$Y \in \zeta_j$, *the overlap of* X *to* Y *is defined in terms of the proximity of their means:*

$$overlap(X,Y) = \begin{cases} 1 - \frac{|\mu(X)-\mu(Y)|}{\sigma(X)} & , |\mu(X) - \mu(Y)| \leq \sigma(X) \\ 0 & , otherwise \end{cases}$$

Definition 5 (Cluster match). *Let* $X \in \zeta_i$, $Y \in \zeta_j$ *be two clusters derived at* $t_i < t_j$ *respectively. Further, let* $\tau \equiv \tau_{match} \in (0.5, 1]$ *be a threshold value.* Y *is "a match for* X *in* ζ_j *subject to* τ*", i.e.* $Y = match_\tau(X, \zeta_j)$*, iff: (i)* Y *has the maximum overlap to* X *among all the clusters in* ζ_j*, i.e.* $overlap(X,Y) = \max_{Y' \in \zeta_j}\{overlap(X,Y')\}$ *and (ii)* $overlap(X,Y) \geq \tau$*. If no such cluster exists for* X *in* ζ_j*, then* $match_\tau(X, \zeta_j) = \emptyset$.

4 Cluster Transitions in MONIC$^+$

For MONIC$^+$, a cluster transition is a change experienced by a cluster that was discovered at the previous timepoint. We use the transition model of MONIC [10]: According to this model, a transition might concern the content and the form of the cluster (internal transition) or rather its relationship to the whole clustering (external transition).

The *external transitions* of cluster $X \in \zeta_i$ with respect to clustering ζ_j discovered at the next timepoint t_j are as follows:
- **Survival:** X survives into $Y \in \zeta_j$, if (a) there is a match Y for it in ζ_j and (b) this match does not contain any further cluster of ζ_i: $X \rightarrow Y$
- **Absorption:** X is absorbed by cluster $Y \in \zeta_j$, if the match Y of X is also match for some other cluster X' of ζ_i: $X \xrightarrow{\subseteq} Y$
- **Split:** X is split into clusters $Y_1, \ldots, Y_p \in \zeta_j$, if each of these clusters overlaps with X for no less than τ_{split} and, when taken together[3], they form a match for X: $X \xrightarrow{\subseteq} \{Y_1, \ldots, Y_p\}$
- **Dissapearance:** X has disappeared, if none of the above cases holds: $X \rightarrow \odot$
The external transitions refer to existing clusters. *Emerging* clusters in ζ_j can be easily detected as those that are not the result of some external transition.

If a cluster survives, *internal transitions* may occur. We categorize internal transitions into changes in size, compactness and location.
- **Size transition:** (a) Cluster shrinks into a smaller cluster: $X \searrow Y$ or (b) expands into a larger cluster: $X \nearrow Y$
- **Compactness transition:** (a) Cluster becomes more compact: $X \xrightarrow{\bullet} Y$ or (b) less compact (more diffuse): $X \xrightarrow{\star} Y$
- **Location transition:** cluster shifts: $X \cdots \rightarrow Y$
- **No change:** $X \leftrightarrow Y$

4.1 Type-Dependent Detection of Transitions

The detection of external transitions in MONIC$^+$ is as in MONIC [10], but some steps must be implemented differently depending on the cluster type. Due

[3] We show later how the "taking all clusters together" is realized for each cluster type.

to space limitations, more details about the transition detection algorithm can be found in the long version of this paper [11].

The observable transitions for each cluster type are depicted in Table 1. All external and internal transitions can be detected for clusters in a metric space (Type A). For clusters defined extensionally (Type B1), compactness and location transitions cannot be observed directly, because concepts like proximity and movement are not defined. However, when one derives the intensional definition of a cluster, both transitions become observable as changes in the cluster density function; we refer to this as Type B1+B2. Conversely, the intensional definition of a cluster (Type B2) does not allow for the detection of splits and absorptions, which in turn can be found by studying the cluster's members (Type B1+B2).

Transition Indicators for Type A Clusters. Let ζ_i, ζ_j be the clusterings at timepoints $t_i < t_j$ and let $X \in \zeta_i$ be the cluster under observation. The transition indicators proposed in Table 2 use the type-specific definition of cluster overlap (Def. 2) and the derived definition of cluster match (Def. 5).

External cluster transitions are detected by computing the area overlap between cluster X and each candidate in ζ_j. To detect a split, we customize the split test of the algorithm [11]. More specifically, we compute the overlap between the area of X and that of all split candidates. Since these candidates cannot overlap, we use the following equation to perform the split test:
$area(X) \cap area(\cup_{u=1}^{p} Y_u) = \sum_{u=1}^{p} area(X) \cap area(Y_u)$

The detection of internal transitions translates into tracing the movements of a cluster in a static metric space. In Table 3, we propose indicators for spherical clusters, as produced by e.g. K-Means and K-Medoids. We can further use indicators for Type B1 and B2 clusters (discussed next).

The first heuristic in Table 3 detects location transitions by checking whether the distance between the centers exceeds a threshold $\tau_{location}$; we normalize this distance on the size of the smallest radius. The second heuristic states that a cluster has become more compact if the average distance from the center was larger in the old cluster than in the new one – subject to a small ε. The third heuristic for clusters becoming less compact is the reverse of the second one.

Transition Indicators for Type B1 Clusters. Let $X \in \zeta_i$ be a cluster found in t_i. To trace its transitions in ζ_j, we consider the indicators proposed in Table 4 for the transitions that can be observed over Type B1 clusters (cf. Table 1).

Size transitions for a cluster X that has survived into Y are traced by comparing the datasets. While the weights used when computing cluster overlap are those valid at timepoint t_j, the size transition heuristics consider the weights of the members of X at the original t_i: The size transition heuristic should reflect the importance of the individual cluster members at t_i.

Transition Indicators for Type B2 Clusters. We consider again a cluster $X \in \zeta_i$. To detect size transitions, we used the heuristic for Type B1 clusters (cf. Table 4). For the other obervable transitions (cf. Table 1), we use the indicators in Table 5. The first one states that a cluster survives if there is a match for it, subject to a $\tau \in (0.5, 1]$ (cf. Def. 5): The indicator demands that $\mu(X)$ and $\mu(Y)$ are closer than half a standard deviation. Since clusters of the same clustering do not overlap, we expect that no more than one cluster of ζ_j satisfies this condition.

An absorption transition for $X \in \zeta_i$ implies finding a $Y \in \zeta_j$ that contains $X, Z \in \zeta_i$. Similarly, a split transition corresponds to finding clusters that contain subsets of X. However, this implies treating the clusters as datasets (Type B1). So, we only consider survival and disappearance for B2-clusters.

To detect compactness transitions, we use the difference of the standard deviations of the clusters X, Y. For location transitions, we use two heuristics that reflect different types of cluster shift: h1 detects shifts of the mean (within half a standard deviation, cf. Def. 5), while h2 traces changes in the skewness $\gamma()$. Heuristic h2 becomes interesting for clusters where the mean has not changed but the distribution exhibits a longer or shorter tail.

5 Experiments

We have tested MONIC$^+$ on a synthetic stream of data records, in which we have imputed cluster transitions. We performed clustering with different algorithms, but here we report on results for B1- and A-clusters.

We used a *data generator* that takes as input the number of data points M, the number of clusters K, as well as the mean and standard deviation of the anticipated members of each cluster. The records were generated around the mean and subject to the standard deviation, following a Gaussian distribution. We fixed the standard deviation to 5 and used a 100×100 workspace for two-dimensional datapoint. The stream was built according to the scenario below.

- **t_1:** Dataset d_1 consists of points around the $K_1 = 5$ centers (20,20), (20, 80), (80, 20), (80, 80), (50, 50).
- **t_2:** Dataset d_2 consists of 40 datapoints, distributed equally across the four corner-groups of d_1 data.
- **t_3:** d_3 consists of 30 points around location (50,40) and 30 points around (50,60).
- **$t_4, \ldots$:** At each of t_4, t_5, t_6, we added 30 points around t_4 :(20,50), t_5 :(20,30) and t_6 :(20,40).

For data ageing, we used a sliding window of size $ws = 2$. Hence, at each timepoint $t_i, i > 1$, the dataset under observation was $D_i = d_i \cup d_{i-1}$.

We have built Type A clusters with K-Means [12]. For Type B2 clusters, we have used Expectation-Maximization (EM) [12], which models clusters as Gaussian distributions; we ignored the distribution information though and treated the clusters as datasets. For K-means, we have defined K to be the optimal number of clusters found by EM. The clusterings found at $t_1, \ldots, t_6$ with EM are shown in Fig. 1. Those found with K-Means are in Fig. 2; they are different from the EM clusters, thus implying also different cluster transitions.

Fig. 1 depicts the clusters at each timepoint but delivers little information about the impact of new data and of data ageing. In Table 6(a), the changes in the population are reflected in the discovered transitions. MONIC$^+$ has correctly mapped the old clusters to the new ones, identifying size transitions, survivals, absorptions and splits. There are also new clusters found at t_4 and t_5.

For Type A clusters, we have used the indicators in Table 2, setting $\tau = 0.5$ and $\tau_{split} = 0.2$. For the size transition, we have used the B1 indicator in Table 4 with $\varepsilon = 0.003$. For the other internal transitions, we have used the indicators for spheres in Table 3 with $\tau_{location} = 0.1$ (location transitions) and $\varepsilon = 0.001$ (compactness transitions). The transitions found by MONIC$^+$ and shown in Table 6(b) reveal that most clusters are unstable, experiencing all types of internal transitions, or they disappear, giving place to new (unstable) clusters. Even in the absence of a visualization (which might be difficult for a real dataset in a multi-dimensional feature space), these transitions indicate the cluster instability and the need for closer inspection of the individual clusters.

6 Conclusion and Outlook

We have presented the framework MONIC$^+$ for the monitoring of cluster transitions over accummulating data. MONIC$^+$ is designed for arbitrary types of clusters, thus making the process of transition detection independent of cluster discovery. MONIC$^+$ employs heuristics that exploit the particular characteristics of different cluster types, such as topological properties for clusters over a metric space (Type A) and descriptors of data distribution for clusters defined as distributions (Type B2). Our first experiments show that the transition model and the detection heuristics can reveal different forms of cluster evolution.

In future work, we intend to design and study dedicated heuristics for specific types of clusters, like spherical ones. We also want to design a more formal evaluation framework: Although there are datasets for the evaluation of stream clustering algorithms, there is no gold standard for the evaluation of evolving clusters upon the stream. Hence, we are considering methods for the generation of appropriate synthetic datasets.

	External	*Internal transitions*		
Cluster type		*Size*	*Compact.*	*Location*
A. metric space	Yes	Yes	Yes	Yes
B1. extensional	Yes	Yes	No	No
B2. intensional	survival	Yes	Yes	Yes
B1+B2.	Yes	Yes	Yes	Yes

Table 1. Observable transitions for each cluster type

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Step	*Transition*	*Indicator*
1	Survival or Absorption	$\exists Y \in \zeta_j : \frac{area(X)\cap area(Y)}{area(X)} \geq \tau$
2	$X \overset{\subseteq}{\to} Y$	$\exists Z \in \zeta_i \setminus \{X\} : \frac{area(Z)\cap area(Y)}{area(Z)} \geq \tau$
3	$X \to Y$	$\nexists Z \in \zeta_i \setminus \{X\} : \frac{area(Z)\cap area(Y)}{area(Z)} \geq \tau$
4	Split	$\exists Y_1, \ldots, Y_p \in \zeta_i :$ $(\forall Y_u : \frac{area(X)\cap area(Y_u)}{area(X)} \geq \tau_{split}) \wedge \frac{area(X)\cap area(\cup_{u=1}^{p} Y_u)}{area(X)} \geq \tau$
5	$X \to \odot$	derived from the above
For survived clusters: $X \to Y$		
Size		B1 indicators & B2 indicators
Compactness		geometry-dependent & B2 indicators
Location		geometry-dependent & B2 indicators

Table 2. Indicators for Type A cluster transitions

Transition	*Indicator*
$X \cdots \to Y$	$\frac{d(center(X),center(Y))}{min\{radius(X),radius(Y)\}} \geq \tau_{location}$
$X \overset{\bullet}{\to} Y$	$avg_{x\in X}(d(x, center(X)) > avg_{y\in Y}(d(y, center(Y)) + \varepsilon$
$X \overset{\star}{\to} Y$	$avg_{y\in Y}(d(y, center(Y)) > avg_{x\in X}(d(x, center(X)) + \varepsilon$

Table 3. Indicators for spherical clusters

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Step	*Transition*	*Indicator*
1	Survival or Absorption	$\exists Y \in \zeta_j : \frac{\sum_{a \in X \cap Y} age(a,t_j)}{\sum_{x \in X} age(x,t_j)} \geq \tau$
2	$X \overset{\subseteq}{\to} Y$	$\exists Z \in \zeta_i \setminus \{X\} : \frac{\sum_{a \in Z \cap Y} age(a,t_j)}{\sum_{z \in Z} age(z,t_j)} \geq \tau$
3	$X \to Y$	$\nexists Z \in \zeta_i \setminus \{X\} : \frac{\sum_{a \in Z \cap Y} age(a,t_j)}{\sum_{z \in Z} age(z,t_j)} \geq \tau$
4	Split	$\exists Y_1, \dots, Y_p \in \zeta_i :$ $(\forall Y_u : \frac{\sum_{a \in X \cap Y_u} age(a,t_j)}{\sum_{x \in X} age(x,t_j)} \geq \tau_{split}) \wedge \frac{\sum_{a \in X \cap (\cup_{u=1}^{p} Y_u)} age(a,t_j)}{\sum_{x \in X} age(x,t_j)} \geq \tau$
5	$X \to \odot$	derived from the above
	Size	
6	$X \nearrow Y$	$\sum_{y \in Y} age(y,t_j) > \sum_{x \in X} age(x,t_i) + \varepsilon$
7	$X \searrow Y$	$\sum_{x \in X} age(x,t_i) > \sum_{y \in Y} age(y,t_j) + \varepsilon$

Table 4. Indicators for Type B1 cluster transitions

Step	*Transition*	*Indicator*
1	$X \to Y$	$\exists Y \in \zeta_j : 1 - \frac{\lvert\mu(X) - \mu(Y)\rvert}{\sigma(X)} \geq \tau$
2	$X \to \odot$	negation of the above
	Size	B1 indicators in Table 4
	$X \cdots \to Y$	h1. $\lvert\mu(X) - \mu(Y)\rvert > \tau_{h1}$ h2. $\lvert\gamma(X) - \gamma(Y)\rvert > \tau_{h2}$ (cf. Eq. **??** below)
	$X \overset{\bullet}{\to} Y$	$\sigma(Y) < \sigma(X) + \varepsilon$
	$X \overset{\star}{\to} Y$	$\sigma(X) < \sigma(Y) + \varepsilon$

Table 5. Indicators for Type B2 cluster transitions

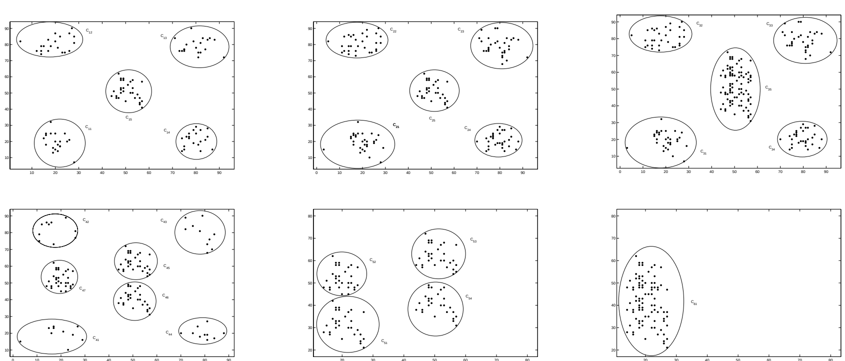

Fig. 1. Type B2 clusters at timepoints t_1, t_2, t_3, t_4 and t_5, t_6

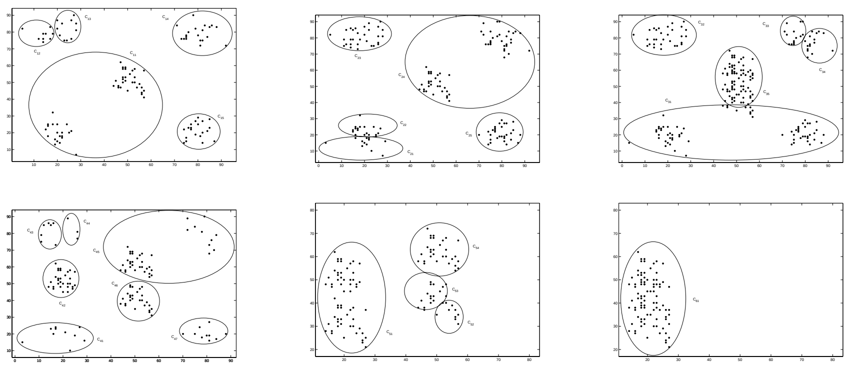

Fig. 2. Type A clusters at timepoints t_1, t_2, t_3, t_4 and t_5, t_6

	Type B2			**Type A**		
t_2	$C_{11} \nearrow C_{21}$	$C_{12} \nearrow C_{22}$ $C_{14} \nearrow C_{24}$	$C_{13} \nearrow C_{23}$ $C_{15} \rightarrow C_{25}$	$C_{11} \rightarrow \odot$ $C_{14} \rightarrow \odot$	$C_{12} \xrightarrow{\subseteq} C_{23}$ $C_{15} \cdots \overset{\bullet}{\rightarrow}\nearrow C_{25}$	$C_{13} \xrightarrow{\subseteq} C_{23}$
t_3	$C_{21} \rightarrow C_{31}$	$C_{22} \rightarrow C_{32}$ $C_{24} \rightarrow C_{34}$	$C_{23} \rightarrow C_{33}$ $C_{25} \nearrow C_{25}$	$C_{21} \rightarrow \odot$ $C_{24} \rightarrow \odot$	$C_{22} \rightarrow \odot$ $C_{25} \rightarrow \odot$	$C_{23} \rightarrow C_{32}$
t_4	$C_{31} \rightarrow \odot$ $C_{34} \rightarrow \odot$	$C_{32} \rightarrow \odot$ $C_{35} \xrightarrow{\subseteq} \{C_{45}, C_{46}\}$	$C_{33} \rightarrow \odot$	$C_{31} \cdots \overset{\bullet}{\rightarrow}\searrow C_{46}$ $C_{34} \rightarrow \odot$	$C_{32} \xrightarrow{\subseteq} \{C_{43}, C_{44}\}$ $C_{35} \cdots \overset{\star}{\rightarrow}\searrow C_{45}$	$C_{33} \rightarrow \odot$
t_5	$C_{41} \rightarrow \odot$ $C_{44} \rightarrow \odot$ $C_{45} \rightarrow C_{53}$	$C_{42} \rightarrow \odot$ $C_{46} \rightarrow C_{54}$	$C_{43} \rightarrow \odot$ $C_{47} \rightarrow C_{52}$	$C_{41} \rightarrow \odot$ $C_{44} \rightarrow \odot$ $C_{46} \xrightarrow{\subseteq} \{C_{52}, C_{53}\}$	$C_{42} \cdots \overset{\star}{\rightarrow}\nearrow C_{51}$ $C_{45} \cdots \overset{\bullet}{\rightarrow}\searrow C_{54}$ $C_{47} \rightarrow \odot$	$C_{43} \rightarrow \odot$
t_6	$C_{51} \xrightarrow{\subseteq} C_{61}$	$C_{52} \xrightarrow{\subseteq} C_{61}$	$C_{51} \overset{\bullet}{\rightarrow}\nearrow C_{61}$	$C_{52} \rightarrow \odot$	$C_{53} \rightarrow \odot$	$C_{54} \rightarrow \odot$

Table 6. Transitions for (a) Type B2 clusters – left and (b) Type A clusters – right